Computing Covers of Plane Forests

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Given a set $T = \{T_1, T_2, \ldots, T_m\}$ of $m$ pairwise non-crossing geometric trees with a total of $n$ vertices in general position. The coverage of these trees is the set of all points $p$ in $\mathbb{R}^2$ such that every line through $p$ intersects at least one of the trees.
Previous Work
Previous Work
Previous Work

Beingessner and Smid 2012:
- Coverage can be computed in $O(m^2 n^2)$ time
- Worst case example with coverage of size $\Omega(n^4)$.
- Problem is $\Theta(n^4)$
Previous Work

Is slowness a consequence of bad inputs being “contrived”? Optimization to be had in structure of “real” inputs?
Observations

Coverage of a single tree is its convex hull
Observations

If two coverages overlap, their combined convex hull is covered
The hull-cover

- Compute the convex hull, $CH(T_i)$, of every tree $T_i \in T$
- If any two convex hulls overlap, replace them with their convex hull
- Repeat until all convex hulls computed thusly are disjoint
- Resulting set of convex polygons is the *hull-cover* of $T$
The hull-cover
The hull-cover
The hull-cover
The hull-cover
The hull-cover
Approximation

Does this approximate the coverage?

- A terrible approximation (for already hard inputs)
- A great approximation (for natural inputs)
Computing the hull-cover

Challenges:

- Finding pairwise intersection is fairly expensive
- Computing convex hulls is fairly expensive
Weakly Disjoint Polygons

Let a *weakly disjoint pair* of convex polygons $P$, $Q$ be a pair of convex polygons such that $P \setminus Q$ and $Q \setminus P$ are both connected sets of points, and $P$ does not share a vertex with $Q$. 
Weakly Disjoint Polygons

A pair of polygons that are weakly disjoint, but not disjoint
Weakly Disjoint Polygons

A pair of polygons that are not weakly weakly disjoint
Weakly Disjoint Polygons

Lemma

If two convex polygons $P, Q$ are weakly disjoint, then their boundaries intersect at at most two points.
Weakly Disjoint Polygons
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Weakly Disjoint Polygons
Lemma

*If two convex polygons \( P, Q \) are weakly disjoint, but not disjoint, then one contains a vertex of the other.*
Weakly Disjoint Polygons
Lemma

The convex hulls of two disjoint trees are weakly disjoint.
Weakly Disjoint Polygons
Lemma
Assume $R$ and $S$ are two non-crossing trees whose convex hulls intersect. Then the convex hull of one is strictly inside the other, or there exists a pair of adjacent vertices on the convex hull of one whose visibility is blocked by the other tree.
Blocked or Nested
Shoot and Insert
Ishaque et al. 2012: $n$ pairwise disjoint polygonal obstacles can be preprocessed in $O(n \log n)$ time and space to support $m$ permanent ray shootings in $O((n + m) \log^2 n + m \log m)$ time.
Algorithm
Algorithm
Algorithm
Algorithm
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Algorithm
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Algorithm
Algorithm
Analysis

$O(n \log^2 n)$ time
$O(n \log n)$ space
The End

Thank you!
Questions?