Computing Covers of Plane Forests

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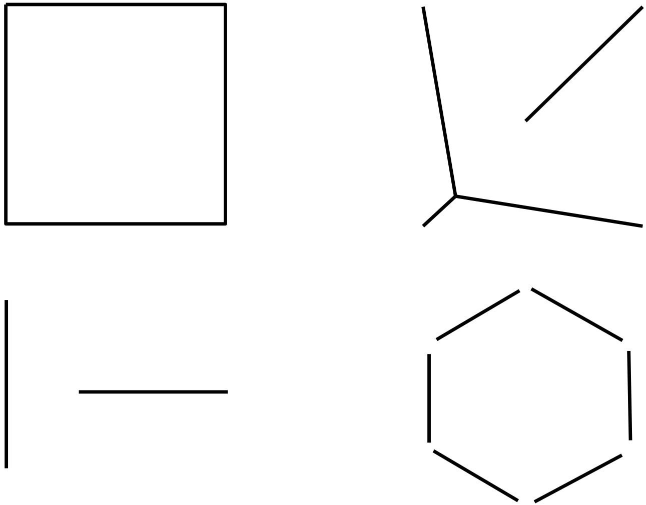
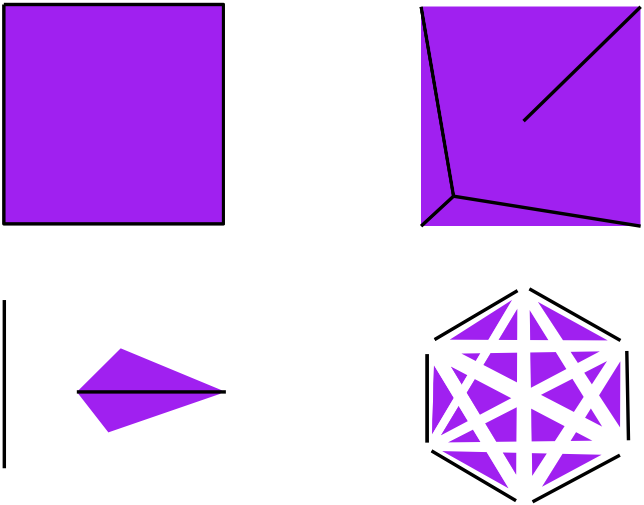
Previous Work

Given a set $T = \{T_1, T_2, \ldots, T_m\}$ of m pairwise non-crossing geometric trees with a total of n vertices in general position. The *coverage* of these trees is the set of all points p in $\mathbb{R}^2$ such that every line through p intersects at least one of the trees.

Previous Work



Previous Work



Previous Work

Beingessner and Smid 2012:

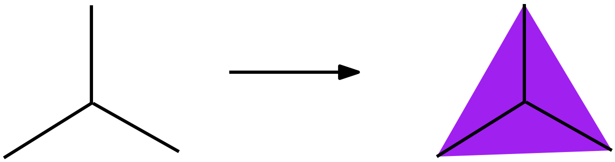
- Coverage can be computed in $O(m^2 n^2)$ time
- Worst case example with coverage of size $\Omega(n^4)$.
- Problem is $\Theta(n^4)$

Previous Work

Is slowness a consequence of bad inputs being “contrived”?
Optimization to be had in structure of “real” inputs?

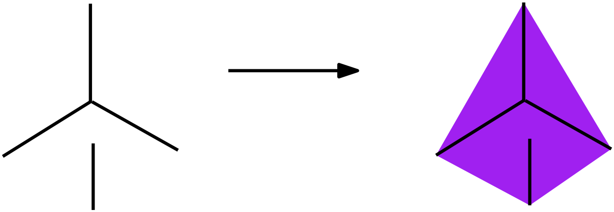
Observations

Coverage of a single tree is it's convex hull



Observations

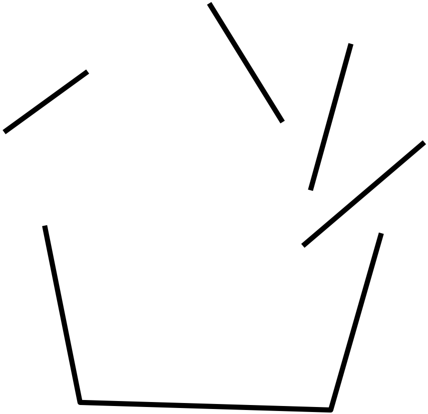
If two coverages overlap, their combined convex hull is covered



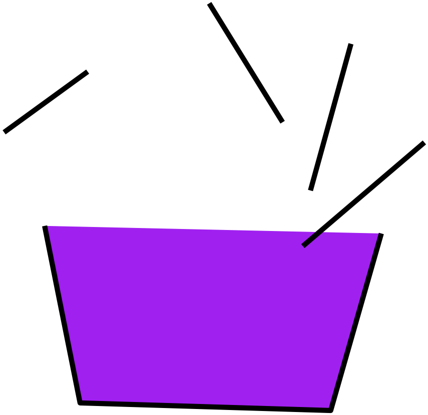
The hull-cover

- Compute the convex hull, $CH(T_i)$, of every tree $T_i \in T$
- If any two convex hulls overlap, replace them with their convex hull
- Repeat until all convex hulls computed thusly are disjoint
- Resulting set of convex polygons is the *hull-cover* of T

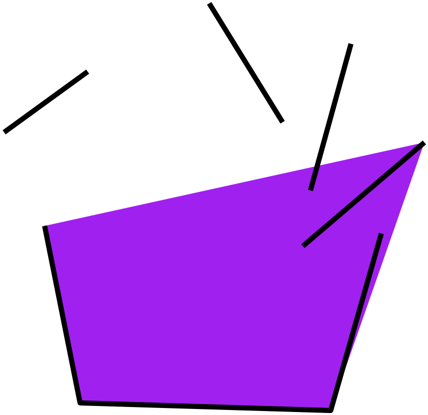
The hull-cover



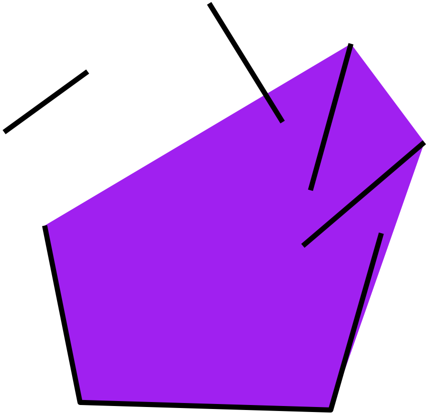
The hull-cover



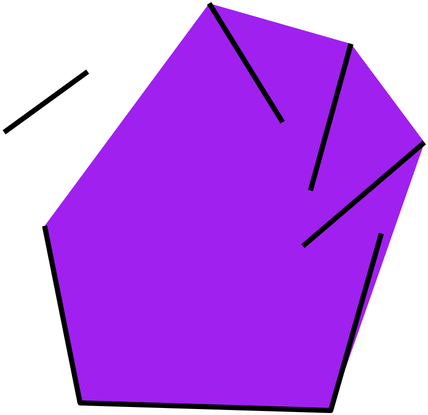
The hull-cover



The hull-cover



The hull-cover



Approximation

Does this approximate the coverage?

- A terrible approximation (for already hard inputs)
- A great approximation (for natural inputs)

Computing the hull-cover

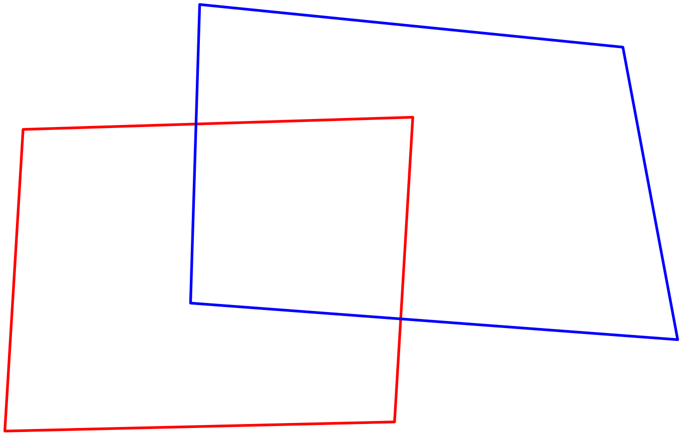
Challenges:

- Finding pairwise intersection is fairly expensive
- Computing convex hulls is fairly expensive

Weakly Disjoint Polygons

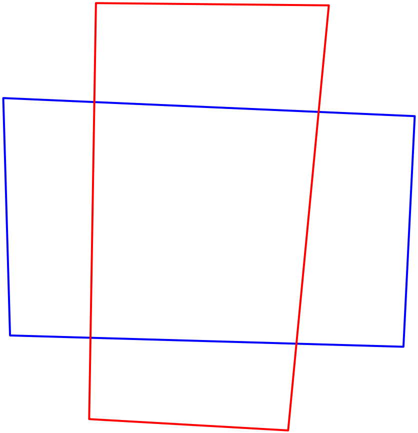
Let a *weakly disjoint pair* of convex polygons P, Q be a pair of convex polygons such that $P \setminus Q$ and $Q \setminus P$ are both connected sets of points, and P does not share a vertex with Q.

Weakly Disjoint Polygons



A pair of polygons that are weakly disjoint, but not disjoint

Weakly Disjoint Polygons



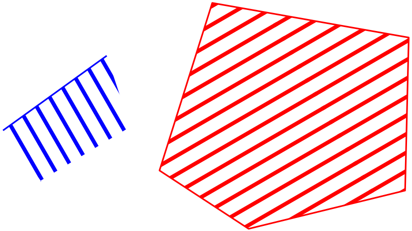
A pair of polygons that are not weakly weakly disjoint

Weakly Disjoint Polygons

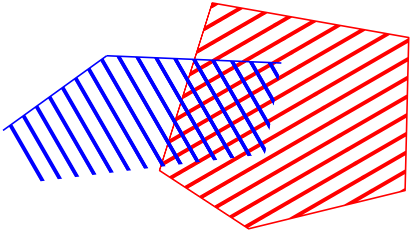
Lemma

If two convex polygons P, Q are weakly disjoint, then their boundaries intersect at at most two points.

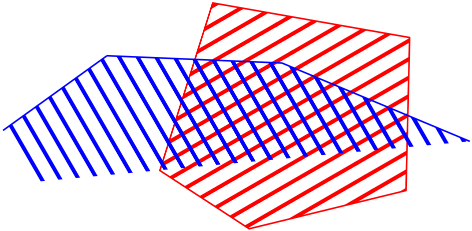
Weakly Disjoint Polygons



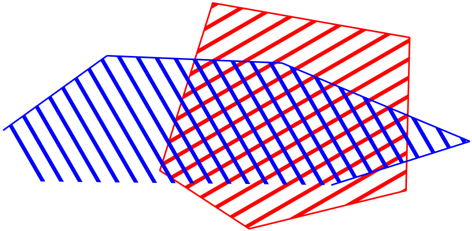
Weakly Disjoint Polygons



Weakly Disjoint Polygons



Weakly Disjoint Polygons

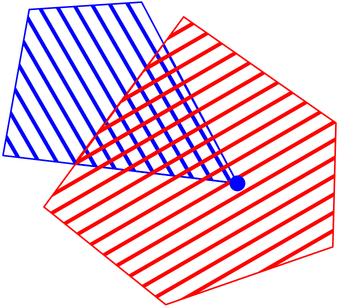


Weakly Disjoint Polygons

Lemma

If two convex polygons P, Q are weakly disjoint, but not disjoint, then one contains a vertex of the other.

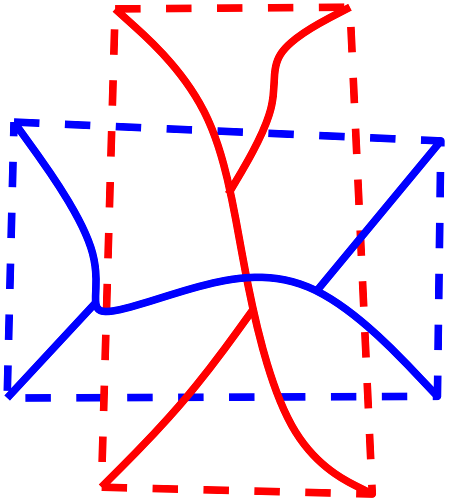
Weakly Disjoint Polygons



Lemma

The convex hulls of two disjoint trees are weakly disjoint.

Weakly Disjoint Polygons

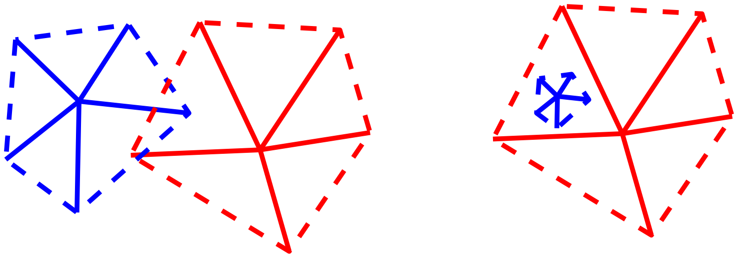


Blocked or Nested

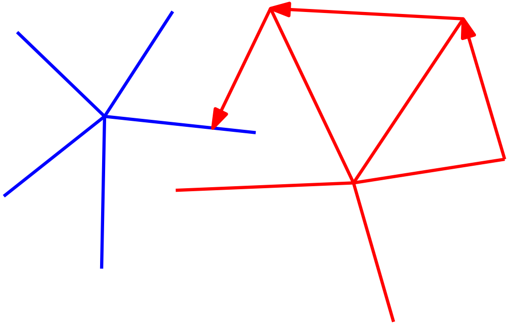
Lemma

Assume R and S are two non-crossing trees whose convex hulls intersect. Then the convex hull of one is strictly inside the other, or there exists a pair of adjacent vertices on the convex hull of one whose visibility is blocked by the other tree.

Blocked or Nested



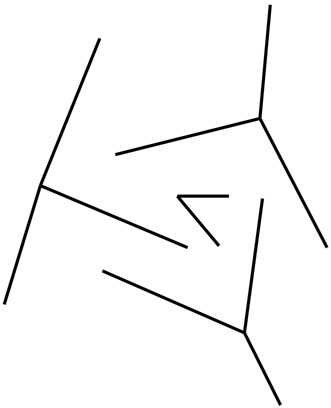
Shoot and Insert



Shoot and Insert

Ishaque et al. 2012: n pairwise disjoint polygonal obstacles can be preprocessed in $O(n \log n)$ time and space to support m permanent ray shootings in $O((n+m)\log^2 n + m \log m)$ time

Algorithm



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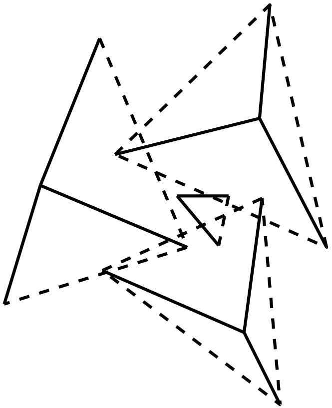

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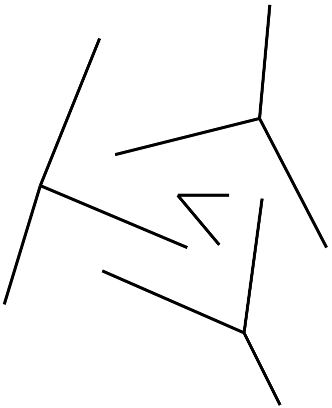

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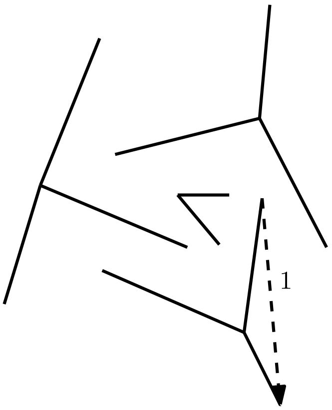

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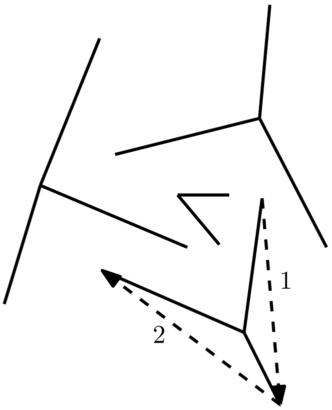

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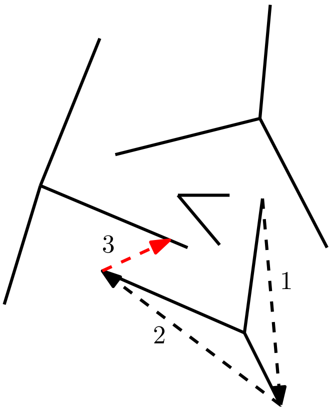

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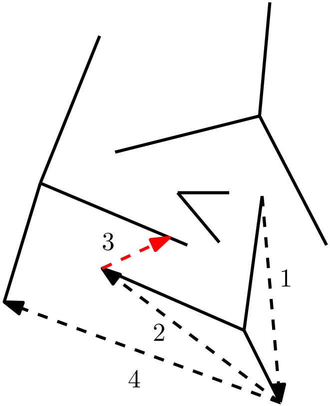

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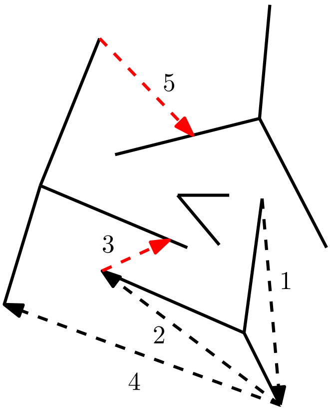

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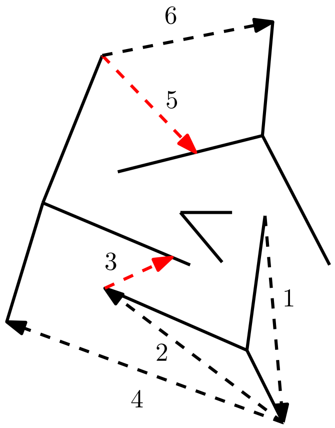

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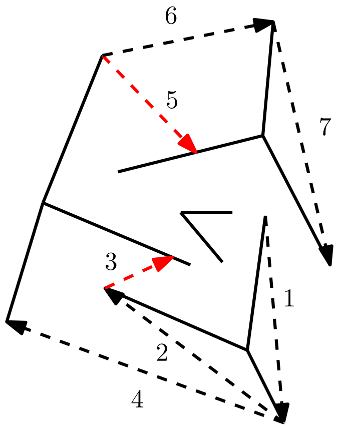

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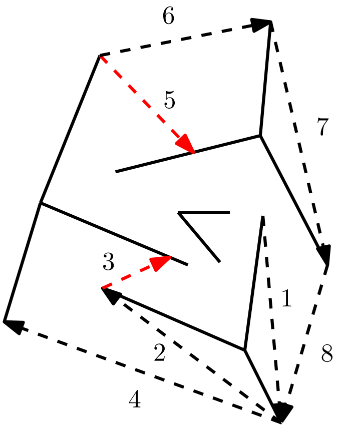

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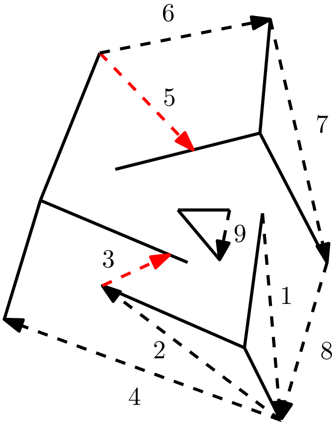

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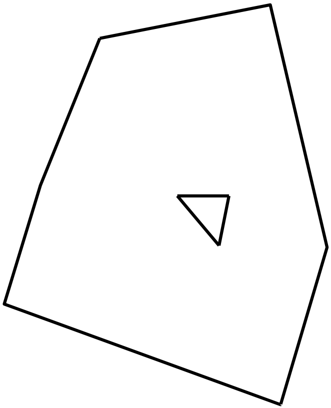

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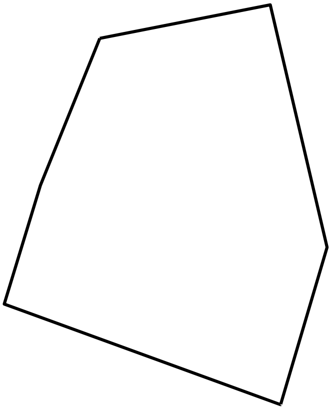

Analysis

$O(n \log^2 n)$ time
$O(n \log n)$ space

The End

Thank you!
Questions?