ON THE SOCIAL PROPERTIES OF A CONIC SECTION AND THE THEORY OF POLEMICAL MATHEMATICS*

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Abstract. This sample file contains text copied verbatim from The Romances of Mathematics [2] about the social properties of a conic, equations to brain waves, social forces, and the laws of political motion. This material was chosen for this example as it is a convenient source of copyright-free text, not for any other reason.

1 Introduction

Most Learned Professors and Students of this University, From the interest manifested in my first lecture, I conclude that my method of investigation has not proved altogether unsatisfactory to you, and I hope ere long to produce certain investigations which will probably startle you, and revolutionize the current thought of the age. The application of mathematics to the study of Social Science and Political Government has curiously enough escaped the attention of those who ought to be most conversant with these matters. I shall endeavour to prove in the present lecture that the relations between individuals and the Government are similar to those which mathematical knowledge would lead us to postulate, and to explain on scientific principles the various convulsions which sometimes agitate the social and political world.

Indeed, by this method we shall be able to prophesy the future of states and nations, having given certain functions and peculiarities appertaining to them, just as easily as we can foretell the exact day and hour of an eclipse of the moon or sun. In order to do this, we must first determine the *social properties of a conic section.*

2 Conic Sections

For the benefit of the unlearned and ignorant, I will first state that a cone is a solid figure described by the revolution of a right-angled triangle about one of the sides containing the right angle, which remains fixed:

$$\{p \in \mathbb{R}^3 : \angle poq = \alpha\} \ .$$

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The fixed side is called the axis of the cone. Conic sections are obtained by cutting the cone by planes. It may easily be proved that if the angle between the cutting plane and the axis be equal to the angle between the axis and the revolving side of the triangle which generates the cone, the section described on the surface of the cone is a parabola; if the former angle be greater than the latter, the curve will be an ellipse; and if less, the section will be a hyperbola.

But the simplest conic section is, of course, a circle, which is formed by a plane at right angles to the axis of the cone; and the simplest circle is that formed by a plane passing through the apex of the cone. All this is simple mathematics; and let beginners consult more elementary treatises than this one to satisfy themselves on these points. But if they will assume these things to be true, they will know quite enough for our present purpose. The simplest conic section of all has been proved to be a *point*. Now, this represents the simplest and original form of society, a *single family*. "It is not good for man to be alone" was the first observation made by the wise Creator upon the rational creature whom He had introduced into Paradise as its lord. Marriage is the rudiment of all social life, from which all others spring, out of which all others are developed. Around the parents' knees soon cluster a group of children, and in their relation to each other we discern the earliest forms of law and discipline—the bonds by which society is held together. When the children grow up, separate households are formed; and then the multiplication of families, the congregating of men together for purposes of security and mutual advantages in division of labour; and thus is gradually formed a state, which is only the development of the family—the king representing the parent, and ruling on the same principle.

3 Chaos and Duality

Mathematically speaking, our plane no longer passes through the apex. The point represented the single family; but keeping the plane horizontal, we move it along the axis, the sections will become *circles*, which represent mathematically the next simplest form of society, where the centre is the seat of government, which is connected with each individual member of the social circle by equal radii. The social property of a circle is that of a monarchical government in its purest and simplest form. The larger the circle becomes (i.e., the further you move the plane from the apex), the greater the distance between the individual and the monarch. Therefore, the more independent the monarchy becomes, and the less influence do individuals possess over the ruling power. Hence, we may infer that as years roll on, the government will become more despotic; but the stability of the country diminished, and probably some individual particle, when sufficiently withdrawn from the attraction of the central head, will begin to revolve on its own account, and spontaneously generate a government of its own. We may, therefore, conclude from mathematical reasoning that an unlimited monarchy, though advantageous for small states, is not a safe form of government for a large or populous country, inasmuch as the people do not derive much benefit from the sovereign; the mutual attraction, which ought to exist in a flourishing state between the ruler and the ruled, is weakened; and the isolation of the monarch tends to make him still more despotic. As a practical example of the truth of the foregoing statement, I may mention the present condition of Russia, which shows that the result of an unlimited monarchy,

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in a large and unwieldy social circle, is such as we should have reasonably expected from mathematical investigations.

Invariably, under the circumstances which I have described, the country will become disorganized; the sovereign will cease to have any power over the people, and the country will become a chaos, without order, influence, or power.

When the centre of a conic section moves along the axis of the curve to infinity, banished by the mutual consent of the individual particles which compose the curve, or the nation, a figure is formed, called a *parabola*. This is the curve which the most erratic bodies in the universe describe in space, as they rush along at a speed inconceivable to human minds, and are supposed to produce all kinds of mischief and injury to the worlds whose courses they wend their way among.

This curve, then, represents the position which the nation assumes when the constituted monarchy, the centre of the system, has been *banished to infinity*. A revolution has occurred; the monarch has been dethroned; and it is not hard to see that the same erratic course which the comet pursues in its flight, is observable with respect to the social system which is represented by a parabola. We observe with eager scrutiny the wanderings of these erratic comets. They appear suddenly with their vapoury tails; sometimes they shine upon us with their soft, silvery light, brilliant as another moon; sometimes they stand afar off in the distant skies, and deign not to approach our steady-going earth, which pursues its regular course day by day, and year by year. Then, after a few days coy inspection of our planet from different points of view, they fly to other remote parts of the universe, and do not condescend to show themselves again for a hundred years or so. Such is the erratic conduct of a heavenly body whose course is regulated by a parabolic curve

$$y = a(x-b)^2 + c \ .$$

We may look for similar eccentric behaviour on the part of a community, nation, or state, whose centre is at infinity, whose constitution has been violently disturbed, and whose monarchy is situated in the far-off regions of unlimited space. The erratic course of Republican rule is proverbial. There is no stability, no regularity. To-day we may observe its brilliancy, which seems to laugh at and eclipse the sombre shining of more steady and enduring worlds; but ere to-morrow's moon has risen, it may have vanished into the regions of eternal night, and we look for its bright shining light in the councils of the nations, but it has ceased to shed its rays, and we are disappointed. Sometimes it is asked, with fear and trembling: "What would be the effect if our earth were to come in contact with the tail of a comet? Should we be destroyed by the collision, and our ponderous world cease to be?" But we are assured that no such disastrous results would follow. We have already passed through the tails of many comets, but we have not discovered any inconvenient change in our ordinary mode of procedure. It is probable that the comet's tail is composed of no solid substance.

We may therefore infer by analogy that a Republican State would not offer any powerful resistance if it were to come into collision with a nation possessing a more settled form of government. A shower of meteoric stones, like passing fireworks, might take place; but beyond that nothing would occur to excite the fear, or arouse the energies of the more

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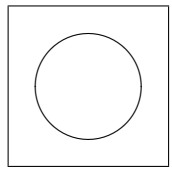

Figure 1: Ceçi n'est pas un cercle.

favoured nation. As an example of the weakness of a Republican State I may mention France. There we see an industrious race of people, endowed with many natural gifts and graces, a country rich and productive; and yet, owing to the unsettled nature of its government, all these natural advantages are neutralized; its course amongst the nations is erratic in the extreme, a spectacle of feeble administration; and it would offer no more resistance to a colliding Power than the empty vacuum of a comet's tail. This example will demonstrate to you the truth of our theory with regard to the instability of a social system which is geometrically represented by a parabolic curve.

4 Ellipses

We will now turn from this picture of insecurity and unrest to another figure which possesses most advantageous social properties. I refer to the ellipse. An ellipse is a curve formed by the section of a cone by a plane surface inclined at an angle to the vertical axis of the cone, greater than the angle between the axis and the generating line. This has nothing to do with the following important result [1, 3]:

Theorem 1 (Appel and Haken 1977)**.** *Every planar graph is* 4*-colourable.*

Proof Sketch. The proof involves a simple but lengthy discharging argument. □

Now, this is a curve which possesses most attractive properties. (See Figure 1.) It is the curve which the earth and other planetary orbs describe around the centre of the solar system, as if nature intended that we should take this figure as a guide in choosing the most advantageous social system. It possesses a centre, C, in view of all the particles which compose the curve, and connected with them by close ties. It has two foci, S and S', fixed points, by the aid of which we may trace the curve.

In the interpretation of this figure, the centre of the curve represents the throne of monarchy. There is no tendency here to revolutionize the State, to banish the ruling power, and institute a Republican form of government; but inasmuch as we saw the weakness of an absolute monarchy in large and populous States, as represented by the circle, the wisdom of an elliptical social system has ordained that there shall be two foci, or houses of representatives of the people, who shall assist in regulating the progress of the nation. Here we have a limited monarchy; the throne is supported by the representatives of the people; and the nearer these foci of the nation are to the centre (i.e., in mathematical language,

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the less the *eccentricity* of the curve), the more perfect the system becomesthe greater the happiness of the community.

In cases where the eccentricity becomes very great, the beauty of the curve is destroyed, and ultimately the ellipse is merged into one straight line. Most learned Professors, here we have a terrible warning of the awful result of too much eccentricity. Whether we regard the life of the nation or of the individual, let all bear in mind this alarming fact, that eccentricity of thought, habit, or behaviour may result, as in the case of this unfortunate ellipse, which once presented such fair and promising proportions to the student's admiring gaze, in the "sinister effacement of a man," or the gradual absorption of a State into an uninteresting thing "which lies evenly between its extreme points."

The great examples of Bacon, of Milton, of Newton, of Locke, and of others, happen to be directly opposed to the popular inference that eccentricity and thoughtlessness of conduct are the necessary accompaniments of talent, and the sure indications of genius. I am indebted to Lacon for that reflection. You may point to Byron, or Savage, or Rousseau, and say, "Were not these eccentric people talented?" "Certainly," I answer; "but would they not have been better and greater men if they had been less eccentric, if they had restrained their caprice, and controlled their passions?" Do not imagine, my young students of this university, that by being eccentric you will therefore become great men and women of genius. The world will not give you credit for being brilliant because you affect the extravagances which sometimes accompany genius. It is apparent that many aspirers to fame and talent are eager to exhibit their eccentricities to the gaze of the world, in order that they may persuade the multitude that they possess the genius of which eccentricity is falsely supposed to be the outward sign.

5 Conclusions

I may remark in passing that the eccentricity of a parabolic curve is always *unity*. What does this prove? You will remember that a Republican State is represented by a parabola. Therefore, however such a nation may strive to alter its condition, and secure a settled form of government, its eccentricity will always remain the same. It will always be erratic, peculiar, unsettled; and this conclusion substantiates our previous proposition with regard to the condition of a social system represented by a parabola.

With regard to other advantages afforded by an elliptical social system, we will defer the consideration of this important subject until my next lecture. For now, we leave you with a question to consider:

Question 1. What is love?

References

[1] K. Appel and W. Haken. Every planar map is four colorable. *Illinois Journal of Mathematics*, 21:439–567, 1977.

[2] P. Hampson. *The Romance of Mathematics*. Oxford Press, Bigg City, 1886.

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[3] N. Robertson, D. P. Sanders, P. Seymour, and R. Thomas. The four-colour theorem. *Journal of Combinatorial Theory, Series B*, 70(1):2–44, 1997.