COMP 3803 — Assignment 1

Due: Thursday January 31, before 11:55pm.

Assignment Policy:

- Your assignment must be submitted as one single PDF file through cuLearn.
- Late assignments will not be accepted.
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

When specifying a finite automaton, it is sufficient to draw the state diagram (because this diagram tells us what are the alphabet, the set of states, the start state, the set of accept states, and the transition function).

Question 1: For each of the following languages, construct a DFA that accepts the language. In all cases, the alphabet is $\{0,1\}$. For each DFA, justify correctness.
(1.1) $\{w \in \{0,1\}^* : w$ starts with 1 and ends with $0\}$.
(1.2) $\{w \in \{0,1\}^* :$ every odd position in w is $1\}$. The positions are numbered $1, 2, 3, \ldots$
(1.3) $\{w \in \{0,1\}^* : w$ has length at least 3 and its third symbol is $0\}$.
(1.4) $\{\epsilon, 0\}$.

Question 2: For each of the following languages, construct an NFA that accepts the language. In all cases, the alphabet is $\{0,1\}$. For each NFA, justify correctness.
(2.1) $\{w : w$ contains the substring 11001$\}$.
(2.2) $\{w : w$ has length at least 2 and does not end with $10\}$.
(2.3) $\{w : w$ begins with 1 or ends with $0\}$.

Question 3: Let A be a language over the alphabet $\Sigma = \{0,1\}$, and let $\bar{A}$ be the *complement* of A. Thus, $\bar{A}$ is the language consisting of all binary strings that are not in A.

Assume that A is a regular language. Let $M = (Q, \Sigma, \delta, q, F)$ be a nondeterministic finite automaton (NFA) that accepts A.

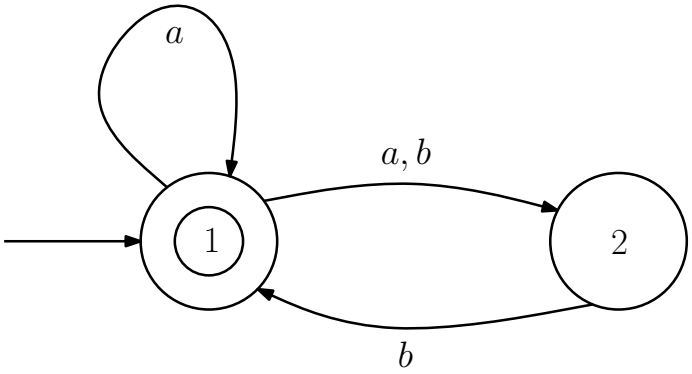
Consider the NFA $N = (Q, \Sigma, \delta, q, \bar{F})$, where $\bar{F} = Q \setminus F$ is the complement of F. Thus, N is obtained from M by turning all accept states into nonaccept states, and turning all nonaccept states into accept states.
(3.1) Is it true that the language accepted by N is equal to $\bar{A}$? Justify your answer.
(3.2) Assume now that M is a deterministic finite automaton (DFA) that accepts A. Define N as above; thus, turn all accept states into nonaccept states, and turn all nonaccept states into accept states. Is it true that the language accepted by N is equal to $\bar{A}$? Justify your answer.

Question 4: Let A and B be two regular languages over the same alphabet Σ. Prove that the difference of A and B, i.e., the language

$$A \setminus B = \{w : w \in A \text{ and } w \notin B\}$$

is a regular language. You may use any result that was proven in class.

Question 5: Use the construction given in class to convert the following NFA to an equivalent DFA.



Question 6: Use the construction given in class to convert the following NFA to an equivalent DFA.