COMP 3803 — Assignment 3

Due: Thursday March 21, before 11:55pm.

Assignment Policy:

- Your assignment must be submitted as one single PDF file through cuLearn.
- Late assignments will not be accepted.
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

Question 1: Give context-free grammars that generate the following languages. For each case, justify your answer.
(1.1) $\{0^{2n}1^n | n \geq 0\}$. The set Σ of terminals is equal to $\{0, 1\}$.
(1.2) $\{w | w$ starts and ends with different symbols$\}$. The set Σ of terminals is equal to $\{0, 1\}$.
(1.3) $\{w |$ w is a palindrome$\}$. The set Σ of terminals is equal to $\{0, 1\}$.
A *palindrome* is a string w having the property that $w = w^R$, i.e., reading w from left to right gives the same result as reading w from right to left. For example, the four binary strings ϵ, 1, 0110, and 10101 are palindromes.
(1.4) $\{a^m b^n c^n | m \geq 0, n \geq 0\}$. The set Σ of terminals is equal to $\{a, b, c\}$.

Question 2: Let L and L' be context-free languages over the same alphabet Σ.
(2.1) Prove that the union $L \cup L'$ of L and L' is also context-free.
(2.2) Prove that the concatenation LL' of L and L' is also context-free.
(2.3) Prove that the star L^* of L is also context-free.

Question 3: Give (deterministic or nondeterministic) pushdown automata that accept the following languages. For each pushdown automaton, start by explaining the algorithm in plain English, then mention the states that you are going to use, then explain the meaning of these states, and finally give the list of instructions.
(3.1) $\{0^{2n}1^n | n \geq 0\}$.
(3.2) $\{ww^R | w \in \{0,1\}^*\}$.
(If $w = w_1 \dots w_n$, then $w^R = w_n \dots w_1$.)

Question 4: Prove that the following languages are not context-free:
(4.1) $\{0^n\, 1\, 0^{2n}\, 1\, 0^{3n} | n \geq 0\}$.
(4.2)

$$\{\ w \in \{a,b,c\}^* \quad | \quad \begin{array}{l} w \text{ contains more } b\text{'s than } a\text{'s } \textbf{and} \\ w \text{ contains more } c\text{'s than } a\text{'s } \}. \end{array}$$

Question 5: In Question 1, you have shown that

$$L = \{a^m b^n c^n | m \geq 0, n \geq 0\}$$

is a context-free language. By a symmetric argument, the language

$$L' = \{a^m b^m c^n | m \geq 0, n \geq 0\}$$

is context-free.
(5.1) Argue that the intersection of two context-free languages is not necessarily context-free. (You may use any result that was proven in class.)
(5.2) Argue that the complement of a context-free language is not necessarily context-free.