Carleton University

Final Examination Fall 2011

DURATION: 3 HOURS — No. of students: 203

Department Name & Course Number: **Computer Science COMP 2805B**
Course Instructor: Michiel Smid

Authorized memoranda: NONE

Students MUST count the number of pages in this examination question paper before beginning to write, and report any discrepancy to the proctor. This question paper has 18 pages (not including the cover page).

This examination question paper MAY NOT be taken from the examination room.

In addition to this question paper, students require:

an examination booklet:	**no**
a Scantron sheet:	**yes**

Student Name:

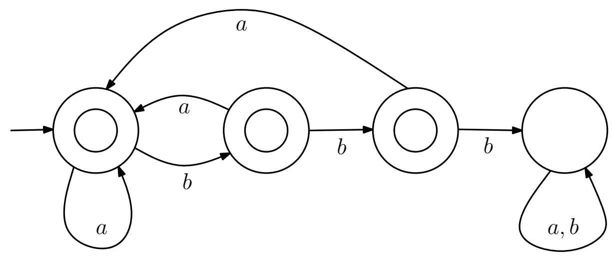
Student Number:

Instructions:

1. This is a closed book exam. No aids, notes, or calculating devices are allowed.
2. All questions must be answered on this examination paper as well as on the scantron sheet.

Marking scheme: Each question is worth 3 marks, except the last one, which is worth 1 mark.

1. Which of the following strings is accepted by this DFA?

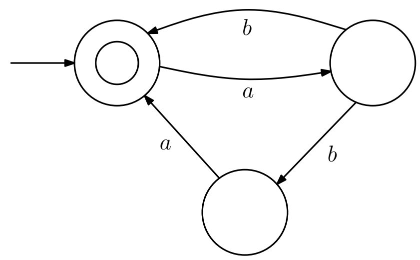


   (a) *ababbbba*
   (b) *bbabbb*
   (c) *bbbbba*
   (d) *ababba*

2. Consider the DFA in the previous question. What is the language of this DFA?

   (a) $\{w \in \{a, b\}^* : \ w$ has an even number of b's$\}$
   (b) $\{w \in \{a, b\}^* : \ w$ has an odd number of b's$\}$
   (c) $\{w \in \{a, b\}^* : \ w$ does not contain the substring $bbb\}$
   (d) $\{w \in \{a, b\}^* : \ w$ contains the substring $bb\}$

3. Which of the following strings is accepted by this NFA?



(a) *baba*

(b) *abaab*

(c) *abba*

(d) *baab*

4. Consider the NFA in the previous question. Which of the following regular expressions describes the language of this NFA?

(a) $(ab \cup aba)^*$

(b) $(ba \cup aba)^*$

(c) $(ab)^*(aba)^*$

(d) None of the above.

5. Which of the following is true?

   (a) Let L be the language of the NFA M. If we turn every accept state into a non-accept state, and turn every non-accept state into an accept state, then we obtain an NFA whose language is the complement $\bar{L}$ of L.

   (b) NFA's with ϵ-transitions can accept languages that cannot be accepted by any NFA without ϵ-transitions.

   (c) For every regular language L, there exists an NFA whose language is L and that has exactly one accept state.

   (d) There exists a language L consisting of a finite number of strings, such that L is not regular.

6. Let M be an NFA with alphabet $\{0, 1\}$ that accepts every binary string. Which of the following is true?

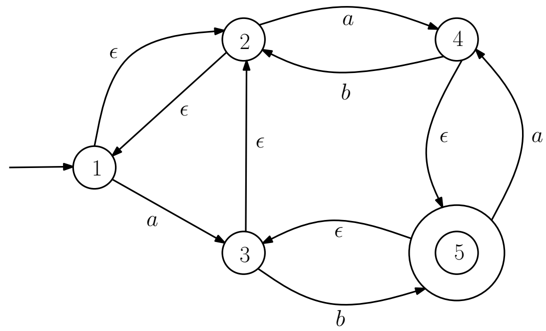
   (a) Every state of M must be an accept state.

   (b) M does not have any accept state.

   (c) The start state of M must be an accept state.

   (d) None of the above.

7. Consider the following NFA.



Assume we convert this NFA to an equivalent DFA (without removing unnecessary states). Consider the following statements:

P : the start state of the DFA is $\{1, 2\}$.
Q : the DFA has 8 accept states.
R : when the DFA is in state $\{5\}$ and reads an a, it switches to the state $\{1, 2, 3, 4, 5\}$.

Which of the following is correct?

(a) P is true, Q is true, R is false.
(b) P is false, Q is false, R is true.
(c) P is true, Q is true, R is true.
(d) P is true, Q is false, R is true.

8. Which of the following strings is in the language that is described by the regular expression $((a \cup b)b^*ab^*)^*$.

(a) $abbbb$
(b) aaa
(c) $bbaa$
(d) $aabbabb$

9. Let L be the language

$$L = \{w \in \{a,b\}^* : \text{ every } a \text{ in } w \text{ is followed by at most one } b\}.$$

Which of the following regular expressions describes the language L?

(a) $b^* \left(aa^*(\epsilon \cup b)\right)^*$
(b) $b^* \left(a^*(\epsilon \cup b)\right)^*$
(c) $b^* \left(a^* \cup a^*b\right)^*$
(d) None of the above.

10. What is the language described by the regular expression

$$c^* \left(a \cup bc^*\right)^*$$

(a) $\{w \in \{a,b,c\}^* : w \text{ contains the substring } ac\}$
(b) $\{w \in \{a,b,c\}^* : w \text{ does not contain the substring } ac\}$
(c) $\{w \in \{a,b,c\}^* : w \text{ contains the substring } ab \text{ or } ba\}$
(d) $\{w \in \{a,b,c\}^* : w \text{ starts with } c\}$

11. In this question, the alphabet is $\{a,b\}$. Consider the following statements:

P : Let R_1 be a regular expression that describes the regular language L_1.
Let R_2 be a regular expression that describes the regular language L_2.
Assume that $\epsilon \in L_1$ and $\epsilon \in L_2$.
Then $(a \cup b)^*$ and $(R_1(a \cup b)^* R_2)^*$ describe different languages.

Q : The regular expressions $(a \cup b)^*$ and $a^*(ba^*)^*$ describe the same language.

Which of the following is correct?

(a) P is true, Q is true.
(b) P is true, Q is false.
(c) P is false, Q is true.
(d) P is false, Q is false.

12. True or false: There exists a regular expression that describes the language $\{a^n b^n : n \geq 0\}$.

(a) True
(b) False

13. True or false: Let L be the language described by the regular expression a^*b^*, and let L' be the language described by the regular expression b^*a^*. Then the regular expression $a^* \cup b^*$ describes the language $L \cap L'$.

    (a) True

    (b) False

14. Recall that $L_1 - L_2$ denotes the difference of the languages L_1 and L_2.
    Thus, $L_1 - L_2 = \{w : w \in L_1, w \notin L_2\}$.
    True or false: For any language L,

    $$LL^* = L^* - \{\epsilon\}.$$

    (a) True

    (b) False

15. Let Σ be a non-empty alphabet. Recall the rules that we used to define regular expressions (RE) over this alphabet:

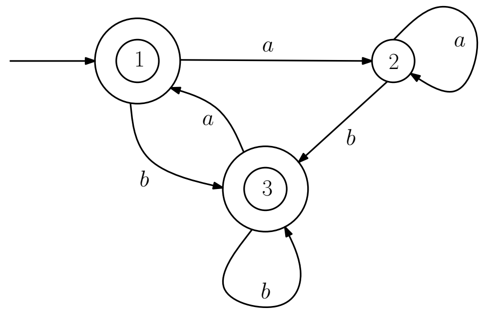
    Rule 1: ϵ is a RE.
    Rule 2: $\emptyset$ is a RE.
    Rule 3: For each $a \in \Sigma$, a is a RE.
    Rule 4: If R_1 and R_2 are RE's, then $R_1 \cup R_2$ is a RE.
    Rule 5: If R_1 and R_2 are RE's, then R_1R_2 is a RE.
    Rule 6: If R is a RE, then R^* is a RE.

    Is the following true or false: We do not need Rule 1, because it follows from Rule 2 and Rule 6.

    (a) True

    (b) False

16. Consider the following DFA.



For each $i = 1, 2, 3$, let L_i be the language of this DFA if we make i the start state. Consider the following statements:

$$P : L_1 = aL_2 \cup bL_3$$
$$Q : L_2 = a^*bL_3$$
$$R : L_3 = b^* \cup b^*aL_1$$

Which of the following is correct?

(a) P is true, Q is true, R is true.

(b) P is true, Q is false, R is true.

(c) P is false, Q is true, R is true.

(d) P is false, Q is true, R is false.

17. Let L be a language consisting of a finite number of binary strings. Which of the following is true?

(a) The Pumping Lemma for regular languages can be applied to L.

(b) The Pumping Lemma for regular languages cannot be applied to L.

18. Consider the context-free grammar $G = (V, \Sigma, R, S)$, where $V = \{S, X, A, B\}$ is the set of variables, $\Sigma = \{a, b\}$ is the set of terminals, S is the start variable, and R consists of the following rules:

$$\begin{aligned} S &\rightarrow aXb \\ X &\rightarrow A|B \\ A &\rightarrow \epsilon|aA \\ B &\rightarrow \epsilon|bB \end{aligned}$$

Which of the following strings is in the language of this grammar?

(a) $aaaaab$

(b) $baaaab$

(c) $abaaaab$

(d) $aaabab$

19. Consider the context-free grammar in the previous question. What is the language of this grammar?

(a) $\{a^n b : n \geq 0\} \cup \{ab^n : n \geq 0\}$

(b) $\{a^n b : n \geq 1\} \cup \{ab^n : n \geq 1\}$

(c) $\{b^n a : n \geq 1\} \cup \{ba^n : n \geq 1\}$

(d) $\{w \in \{a, b\} : \ w \text{ does not contain the substring } ba\}$

20. Consider the context-free grammar $G = (V, \Sigma, R, S)$, where $V = \{S, A\}$ is the set of variables, $\Sigma = \{a, b\}$ is the set of terminals, S is the start variable, and R consists of the following rules:

$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow AAA|a|bA|Ab \end{aligned}$$

Which of the following strings is in the language of this grammar?

(a) $babbab$

(b) bb

(c) $bbbbb$

(d) bba

21. Let A and B be languages such that $A \subseteq B$; thus, A is a subset of B. Assume that the language A is context-free. Which of the following is true?

(a) B must be context-free.

(b) B cannot be context-free.

(c) B may be context-free.

(d) None of the above.

22. Consider the pushdown automaton with tape alphabet $\Sigma = \{a, b\}$, stack alphabet $\Gamma = \{\$, S\}$, set of states $Q = \{q_0, q_1, q_2\}$, start state q_0, and instructions

$$\begin{array}{lllllllll} q_0 a\$ & \rightarrow & q_1 R\$S & \quad q_1 aS & \rightarrow & q_2 R\epsilon & \quad q_2 a\$ & \rightarrow & q_2 R\$ \\ q_0 b\$ & \rightarrow & q_0 R\$ & \quad q_1 bS & \rightarrow & q_0 R\epsilon & \quad q_2 b\$ & \rightarrow & q_2 R\$ \\ q_0 \Box\$ & \rightarrow & q_0 N\$ & \quad q_1 \Box S & \rightarrow & q_1 NS & \quad q_2 \Box\$ & \rightarrow & q_2 N\epsilon \end{array}$$

Recall that, at the start of the computation, the stack contains the symbol \$ (and nothing else). Which of the following strings is accepted by this pushdown automaton?

(a) ϵ

(b) $baba$

(c) $babaab$

(d) $abba$

23. Consider the pushdown automaton in the previous question. What is the language of this pushdown automaton?

(a) $\{w \in \{a, b\}^* : w \text{ contains the substring } bb\}$

(b) $\{w \in \{a, b\}^* : w \text{ does not contain the substring } aa\}$

(c) $\{w \in \{a, b\}^* : w \text{ contains the substring } ba\}$

(d) $\{w \in \{a, b\}^* : w \text{ contains the substring } aa\}$

24. Recall that, for any string $w = w_1 w_2 \ldots w_{n-1} w_n$, we denote by w^R the string obtained by reversing w. Thus, $w^R = w_n w_{n-1} \ldots w_2 w_1$.
Consider the following statements:

P : There exists a deterministic pushdown automaton whose language is $\{wcw^R : w \in \{a,b\}^*\}$. Here, the alphabet is $\{a,b,c\}$.
Q : There exists a nondeterministic pushdown automaton whose language is $\{ww^R : w \in \{a,b\}^*\}$. Here, the alphabet is $\{a,b\}$.

Which of the following is correct?

(a) P is true, Q is true.
(b) P is true, Q is false.
(c) P is false, Q is true.
(d) P is false, Q is false.

25. Let $L = \{a^m b^n : 0 \leq m \leq n \leq 2m\}$. Which of the following is correct?

(a) There exists an NFA whose language is L.
(b) There does not exist a context-free grammar whose language is L.
(c) There exists a non-deterministic pushdown automaton whose language is L.
(d) There does not exist a non-deterministic pushdown automaton whose language is L.

26. Let $A = \{www : w \in \{a,b\}^*\}$. Assume we use the Pumping Lemma to prove that A is not a context-free language. Which of the following strings can be used to obtain a contradiction? (p denotes the pumping length.)

(a) $s = a^p b^p a^p b^p$.
(b) $s = a^p a^p a^p$.
(c) $s = a^p b^p a^p b^p a^p b^p$.
(d) All of the above.

27. Consider the Turing machine with input alphabet $\Sigma = \{a, b\}$, tape alphabet $\Gamma = \{a, b, \Box\}$, set of states $Q = \{q, q_{accept}, q_{reject}\}$, start state q, and instructions

$$\begin{aligned} qa &\rightarrow qaR \\ qb &\rightarrow qbL \\ q\Box &\rightarrow q_{accept} \end{aligned}$$

Recall that, at the start of the computation, the tape head is at the leftmost symbol of the input string. Which of the following regular expressions describes the language of this Turing machine?

(a) $a^* \cup b(a \cup b)^*$

(b) $a^*b \cup b(a \cup b)^*$

(c) $ba^* \cup b(a \cup b)^*$

(d) $(a \cup b)^*$

28. Consider the Turing machine with input alphabet $\Sigma = \{a, b\}$, tape alphabet $\Gamma = \{a, b, \Box\}$, set of states $Q = \{q_0, q_1, q_{accept}, q_{reject}\}$, start state q_0, and instructions

$$\begin{array}{lcl} q_0 a & \rightarrow & q_1 a R \\ q_0 b & \rightarrow & q_0 b R \\ q_0 \Box & \rightarrow & q_{reject} \\ q_1 a & \rightarrow & q_{accept} \\ q_1 b & \rightarrow & q_0 b R \\ q_1 \Box & \rightarrow & q_{reject} \end{array}$$

Recall that, at the start of the computation, the tape head is at the leftmost symbol of the input string. Which of the following strings is accepted by this Turing machine?

(a) ϵ

(b) $bbabaab$

(c) $ababab$

(d) $abba$

29. Consider the Turing machine in the previous question. What is the language of this Turing machine?

(a) $\{w \in \{a, b\}^* : w$ contains the substring $aa\}$

(b) $\{w \in \{a, b\}^* : w$ contains the substring $ab\}$

(c) $\{w \in \{a, b\}^* : w$ contains the substring $ba\}$

(d) $\{w \in \{a, b\}^* : w$ contains the substring $bb\}$

30. We proved in class that the Halting Problem is undecidable. Which proof technique did we use?

    (a) We used a proof by contradiction.
    (b) We used the Pumping Lemma for decidable languages.
    (c) We used induction on the number of states of the Turing machine that decides the Halting Problem.
    (d) We used Einstein's Theory of Relativity.

31. Let M be a Turing machine with input alphabet $\{0, 1\}$ and let w be a binary string.
    We construct a new Turing machine T_{Mw} which takes as input an arbitrary binary string x:

    Turing machine $T_{Mw}(x)$:
    if $x = 0^n 1^n$ for some $n \geq 0$
    then terminate in the accept state
    else run M on the input string w;
        if M terminates in the accept state
        then terminate in the accept state
        else if M terminates in the reject state
            then terminate in the reject state
            endif
        endif
    endif

    Recall that $L(T_{Mw})$ denotes the set of all binary strings x that are accepted by the Turing machine T_{Mw}. Consider the following statements:

    P : If M accepts w, then $L(T_{Mw})$ is not a regular language.
    Q : If M does not accept w, then $L(T_{Mw})$ is a regular language.

    Which of the following is correct?

    (a) P is true, Q is true.
    (b) P is true, Q is false.
    (c) P is false, Q is true.
    (d) P is false, Q is false.

32. Consider the following statements:

    P : For every decidable language L, the complement of L is also decidable.
    Q : For every enumerable language L, the complement of L is also enumerable.

    Which of the following is correct?

    (a) P is true, Q is true.
    (b) P is true, Q is false.
    (c) P is false, Q is true.
    (d) P is false, Q is false.

33. True or false: Let A be an enumerable language. Then there *exists* a Turing machine M such that for any input string w,

$$w \in A \text{ if and only if } M \text{ terminates on input } w.$$

    (a) True
    (b) False

34. How do you feel?

    (a) I need a beer.