COMP 3804 — Assignment 4

Due: Monday April 8, 23:59.

Assignment Policy:

- Your assignment must be submitted as one single PDF file through Brightspace.

> Use the following format to name your file:
>
> `LastName_StudentId_a4.pdf`

- **Late assignments will not be accepted. I will not reply to emails of the type "my internet connection broke down at 23:57" or "my scanner stopped working at 23:58", or "my dog ate my laptop charger".**
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

Question 1: Write your name and student number.

Question 2: The *set cover problem* is defined as follows:

$$\text{SetCover} = \{(S, n, A_1, A_2, \ldots, A_m, K) : \begin{array}{l} S \text{ is a set of size } n, \text{ each } A_i \text{ is a subset of } S, \\ \exists I \subseteq \{1, 2, \ldots, m\} \text{ such that} \\ |I| = K \text{ and } \cup_{i \in I} A_i = S \end{array}\}.$$

Prove that SetCover is in **NP**.

Question 3: *Los Tabernacos* is a famous poutine restaurant in Playa del Carmen, Mexico. The owners want to advertize their restaurant to all people ("users") on Instagram. For a given integer K, they ask K users to post a picture of the restaurant[1] on their account.

All users follow the Instagram etiquette: If user u posts a picture, then all users who follow u post a copy of this picture.

Can the owners of Los Tabernacos choose K users such that all Instagram users post a picture of the restaurant?

- Formulate this problem as a decision problem LosTabernacos on a graph.
- Prove that LosTabernacos $\leq_P$ SetCover, i.e., in polynomial time, LosTabernacos can be reduced to SetCover.

Question 4: Let $G = (V, E)$ be an undirected graph. A *Hamilton cycle* is a cycle in G that contains every vertex exactly once. A *Hamilton st-path* is a path in G between the vertices s and t that contains every vertex exactly once.

Consider the problems

$$\text{HamiltonCycle} = \{G : \text{ graph } G \text{ contains a Hamilton cycle}\}$$

and

$$\text{HamiltonPath} = \{(G, s, t) : \text{ graph } G \text{ contains an } st\text{-Hamilton path}\}.$$

- Prove that HamiltonCycle $\leq_P$ HamiltonPath, i.e., in polynomial time, HamiltonCycle can be reduced to HamiltonPath.
- Prove that HamiltonPath $\leq_P$ HamiltonCycle, i.e., in polynomial time, HamiltonPath can be reduced to HamiltonCycle.

Question 5: In the *longest path problem*, we are given an undirected graph $G = (V, E)$ in which each edge has a positive weight, two vertices s and t, and a number L. The question is whether or not G contains an st-path (i.e., a path between s and t) of length at least L. In such a path, any vertex cannot be visited more than once.

[1] and offer them free poutine

$$\textsc{LongestPath} = \{(G, s, t, L) : \text{ graph } G \text{ contains an } st\text{-path of length at least } L\}.$$

Prove that HamiltonCycle $\leq_P$ LongestPath, i.e., in polynomial time, HamiltonCycle can be reduced to LongestPath.

Question 6: A Boolean formula φ, in the variables $x_1, x_2, \ldots, x_n$, is in *three conjunctive normal form* (3CNF), if it is of the form

$$\varphi = C_1 \wedge C_2 \wedge \ldots \wedge C_m,$$

where each *clause* C_i, $1 \leq i \leq m$, is of the form

$$C_i = l_1^i \vee l_2^i \vee l_3^i.$$

Each l_j^i is a *literal*, which is either a variable or the negation of a variable.

The *three-satisfiability problem* is defined as follows:

$$\textsc{3Sat} = \{\varphi : \varphi \text{ is in 3CNF-form and is satisfiable}\}.$$

A *vertex cover* of an undirected graph $G = (V, E)$ is a subset X of V such that for each edge $\{u, v\}$ in E, at least one of u and v is in X.

The *vertex cover problem* is defined as follows:

$$\textsc{VertexCover} = \{(G, K) : \text{ graph } G \text{ contains a vertex cover of size } K\}.$$

Prove that 3Sat $\leq_P$ VertexCover, i.e., in polynomial time, 3Sat can be reduced to VertexCover.