COMP4804 Important Facts

Union of Events and Boole’s Inequality. For any events $A$ and $B$

$$\Pr\{A \text{ or } B\} = \Pr\{A\} + \Pr\{B\} - \Pr\{A \text{ and } B\}$$

$$\leq \Pr\{A\} + \Pr\{B\}.$$  

Conditional Probability. 

$$\Pr\{A \mid B\} = \frac{\Pr\{A \text{ and } B\}}{\Pr\{B\}}$$

Another useful way of writing this is

$$\Pr\{A \text{ and } B\} = \Pr\{A \mid B\} \Pr\{B\}.$$ (1)

Independence. We say that $A$ and $B$ are independent if and only if 

$$\Pr\{A \mid B\} = \Pr\{A\}$$

If $A$ and $B$ are independent then (1) becomes

$$\Pr\{A \text{ and } B\} = \Pr\{A\} \Pr\{B\}$$ (Only if $A$ and $B$ are independent!)

Expected Value. For a random variable $X$

$$\mathbb{E}[X] = \sum_x x \Pr\{X = x\}.$$ 

Linearity of Expectation. For any random variables $X$ and $Y$

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y].$$

More generally, for any random variables $X_1, \ldots, X_n$

$$\mathbb{E}\left[\sum_{i=1}^n X_i\right] = \sum_{i=1}^n \mathbb{E}[X_i].$$

Linearity of expectation, in combination with indicator variables, is extremely useful for things we can count.

Markov’s Inequality. For any non-negative random variable $X$, 

$$\Pr\{X > t\mathbb{E}[X]\} \leq \frac{1}{t}.$$ 

Bernoulli and Binomial Random Variables. A Bernoulli($p$) random variable is a random variable that is equal to 1 with probability $p$ and 0 with probability $1-p$. If $X$ is a Bernoulli($p$) random variable then $\mathbb{E}[X] = p$. A binomial($p,n$) random variable is the sum of $n$ independent Bernoulli($p$) random variables. If $B$ is a Bernoulli($p,n$) random variable then $\mathbb{E}[B] = pn$. Also, don’t forget Chernoff’s bounds:

$$\Pr\{B \geq (1 + \epsilon)np\} \leq e^{-\epsilon^2 np/3}$$

and

$$\Pr\{B \leq (1 - \epsilon)np\} \leq e^{-\epsilon^2 np/2}$$

Beware: Chernoff’s bounds are only for binomial random variables. In particular you must make sure that the $X_i$s are independent!