New and Improved Spanning Ratios for Yao Graphs

Luis Barba[1,2] Prosenjit Bose[1] Mirela Damian[3] Rolf Fagerberg[4]
Wah Loon Keng[5] Joseph O'Rourke[6] André van Renssen[1]
Perouz Taslakian[7] Sander Verdonschot[1] Ge Xia[5]

[1]Carleton University

[2]Université Libre de Bruxelles

[3]Villanova University

[4]University of Southern Denmark

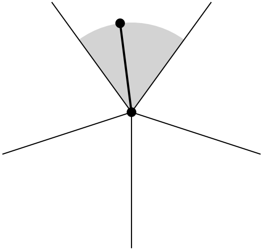
[5]Lafayette College

[6]Smith College [7]American University of Armenia

30th Annual Symposium on Computational Geometry

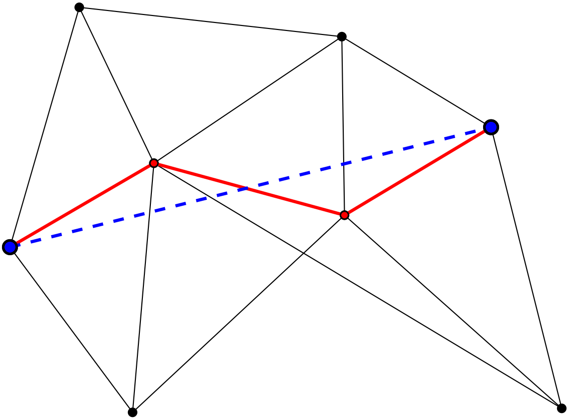
Yao-graphs

- Partition plane into k cones
- Add edge to closest vertex in each cone



Geometric Spanners

- Graphs with short detours between vertices
- For every u and w, there is a path with length $\leq t \cdot |uw|$



Previous Work

$k > 8$ $(1+\varepsilon)$ (Althöfer *et al.*, 1993)

$k > 8$ $\dfrac{1}{\cos\theta - \sin\theta}$ (Bose *et al.*, 2004)

$k > 6$ $\dfrac{1}{1 - 2\sin\frac{\theta}{2}}$ (Bose *et al.*, 2010)

Previous Work

$k > 6$	$1/(1 - 2\sin(\theta/2))$	(Bose *et al.*, 2010)
Y_6	**??**	
Y_5	**??**	
Y_4	**??**	
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$k > 6$	$1/(1 - 2\sin(\theta/2))$	(Bose *et al.*, 2010)
$k > 3$ and odd	$1/(1 - 2\sin(3\theta/8))$	
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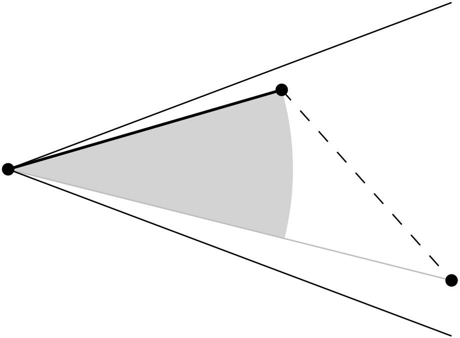
Odd Yao graphs

- Basic lemma (used for Yao graphs with $k > 6$)



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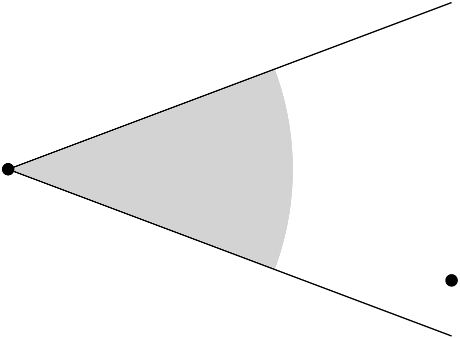

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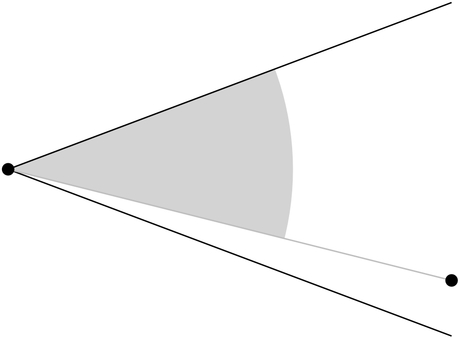
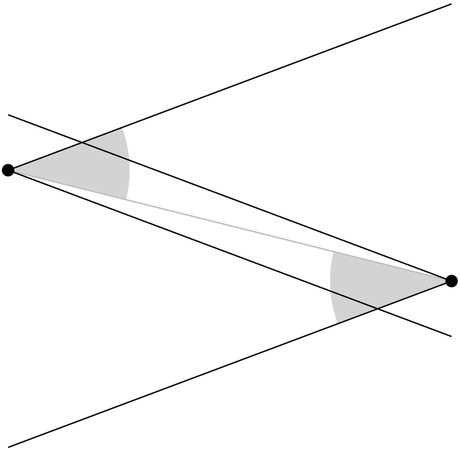


Odd Yao graphs

- Even number of cones: Increasing one angle also increases the other



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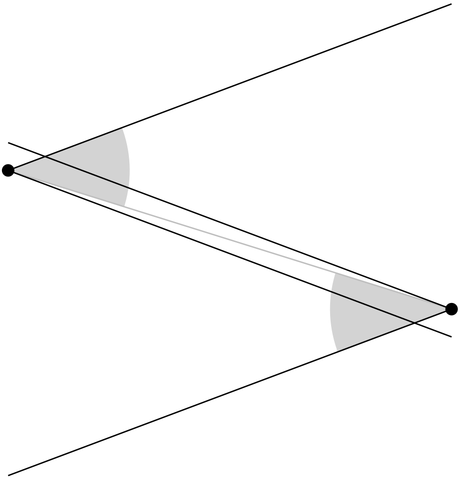
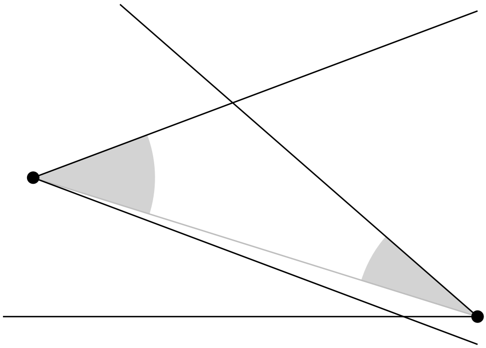


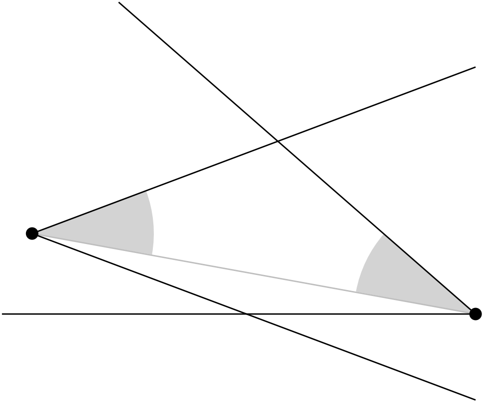
Odd Yao graphs

- Odd number of cones: Increasing one angle decreases the other



Odd Yao graphs

- Odd number of cones: Worst case occurs for $3\theta/4$



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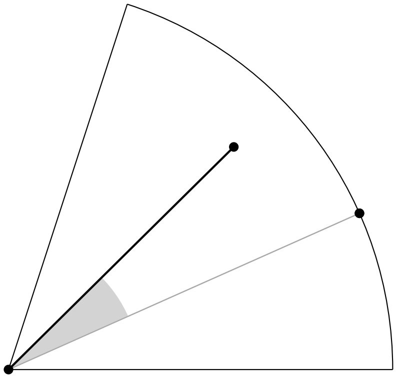
- ~~Always~~ Strategically apply basic lemma
- Handle remaining cases

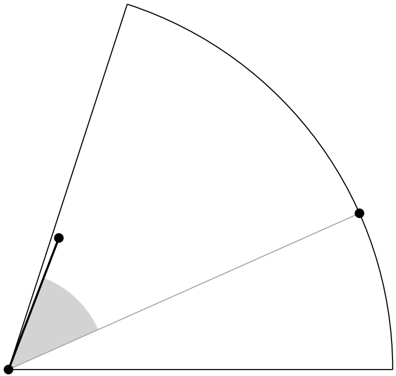
Improvements for Y_5

- What if we only apply the lemma for small angles?



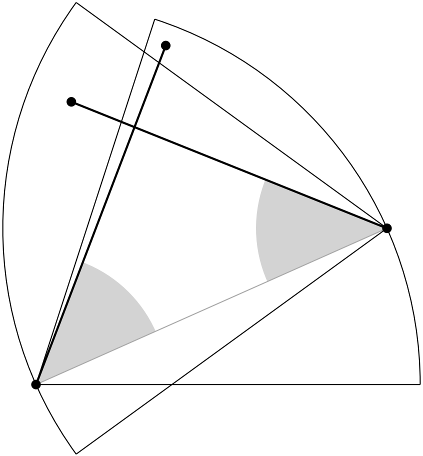
Improvements for Y_5

- If the edge is very short, we're still okay



Improvements for Y_5

- Case 1: Both edges are long and they cross



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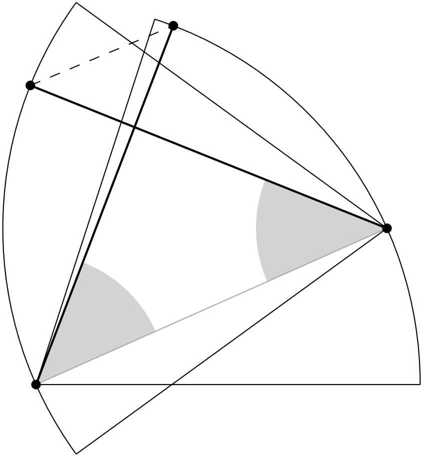

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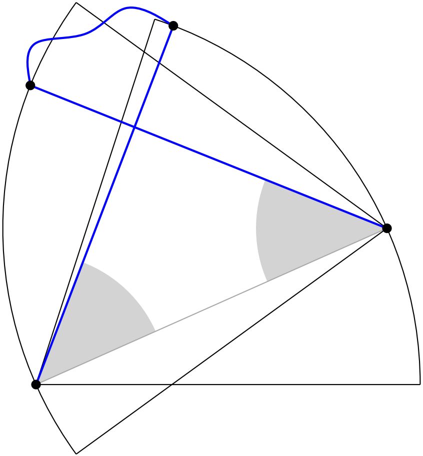

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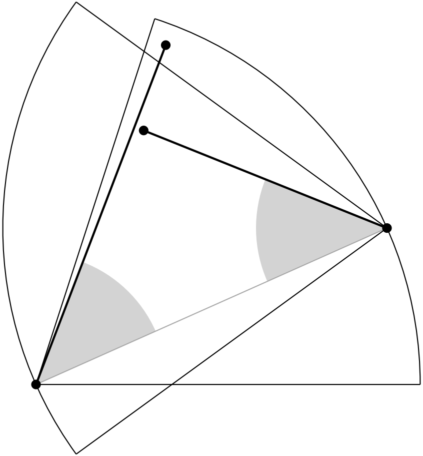
- Case 2: Both edges are long and do not cross



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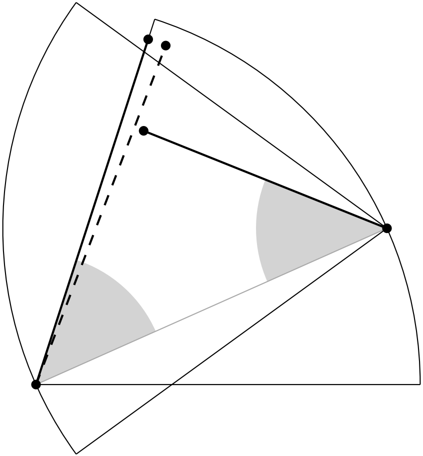

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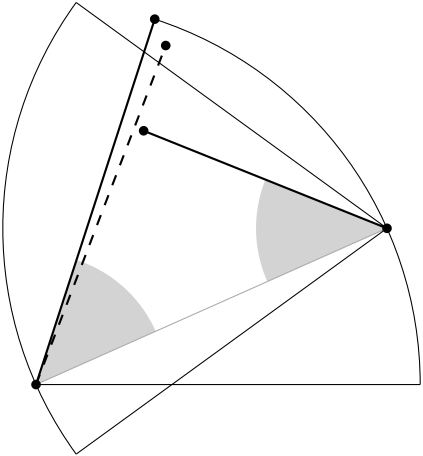

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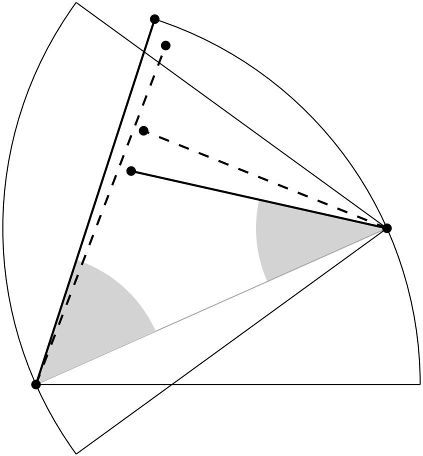

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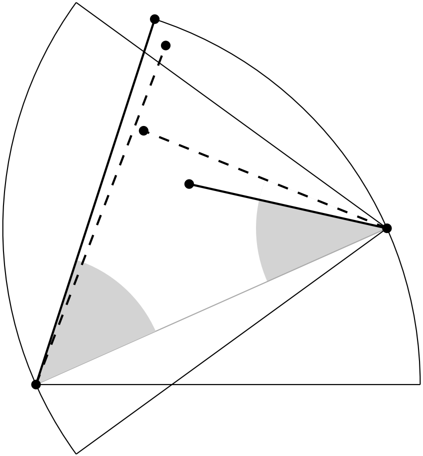

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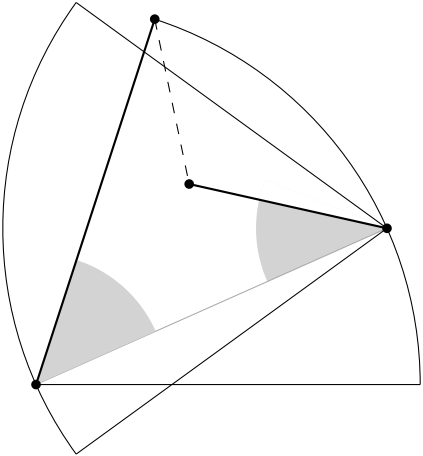

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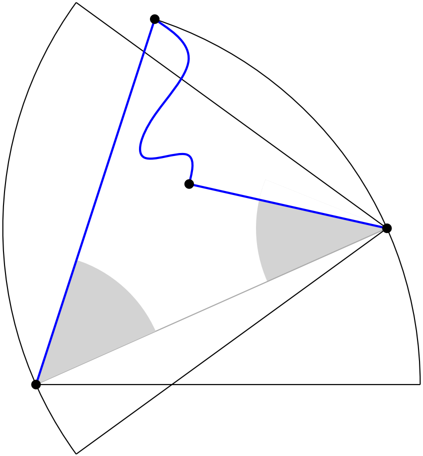

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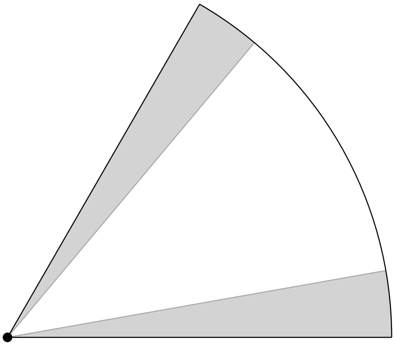
Spanning ratio of Y_6

Same general idea:

- Strategically apply basic lemma
- Handle remaining cases

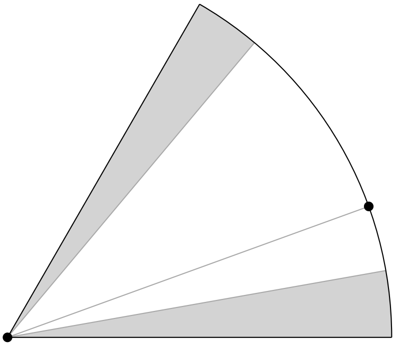
Spanning ratio of Y_6

- Split cone into center and margins



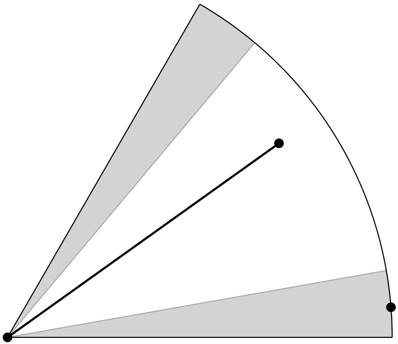
Spanning ratio of Y_6

- Destination in center → Apply basic lemma



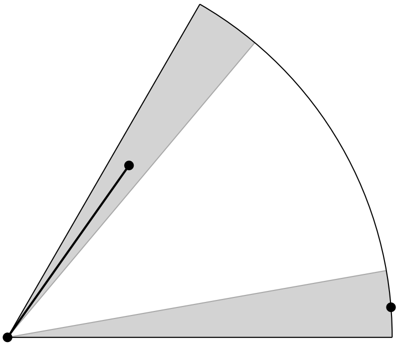
Spanning ratio of Y_6

- Closest in center → Apply basic lemma



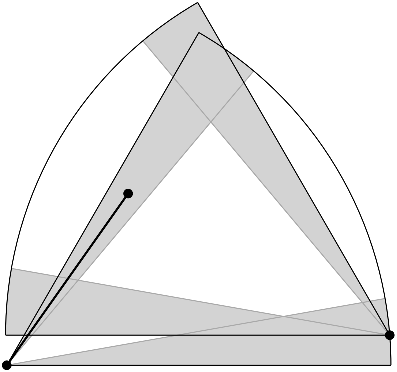
Spanning ratio of Y_6

- Closest in center $\rightarrow$ Apply basic lemma



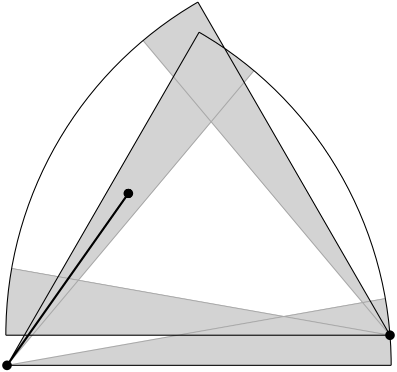
Spanning ratio of Y_6

- Look from the other side



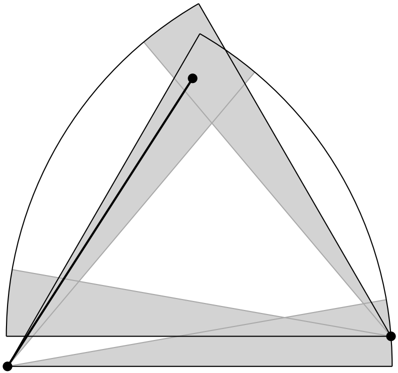
Spanning ratio of Y_6

- Closest in center → Apply (variation of) basic lemma



Spanning ratio of Y_6

- If closest lies in both margins,



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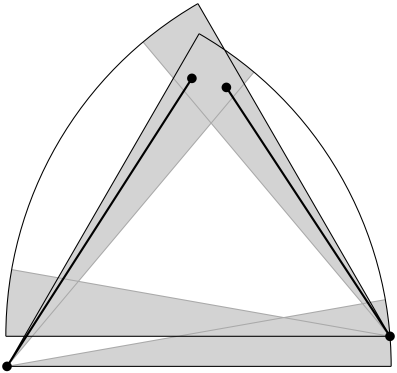
- If closest lies in both margins, consider the other closest



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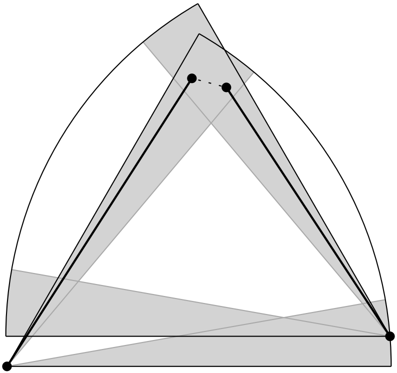

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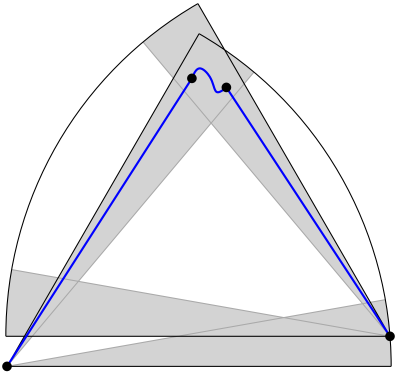
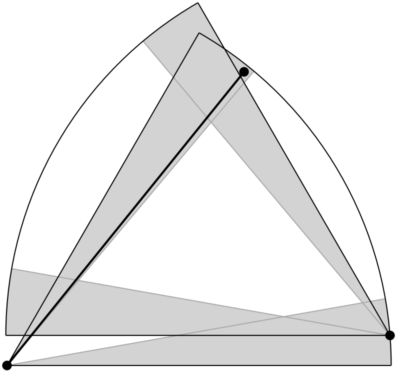


Spanning ratio of Y_6

- A few more cases...



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- Improved lower bounds
- Competitive routing

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Questions?