Rectangle-of-influence triangulations

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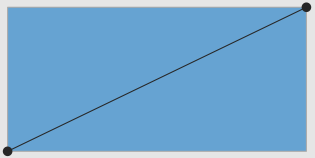
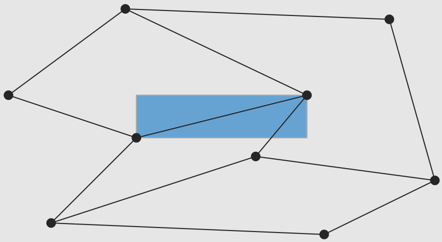
RI-Edges

- An edge is RI if its supporting rectangle (smallest axis-aligned bounding box) is empty of (other) points



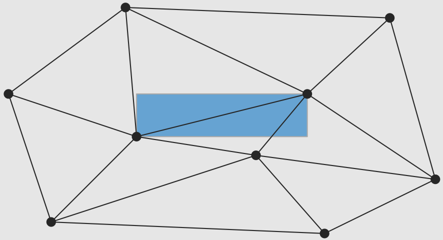
RI-Drawings

- Drawing of a graph where all edges are RI
- Well-studied in Graph Drawing community



RI-Triangulations

- All internal faces are triangles
- Maximal



RI-Problems

1. RI-triangulating a polygon
2. RI-triangulating a point set
3. Flipping one RI-triangulation to another
4. Flipping a triangulation to an RI-triangulation

RI-Problems

1. **RI-triangulating a polygon**
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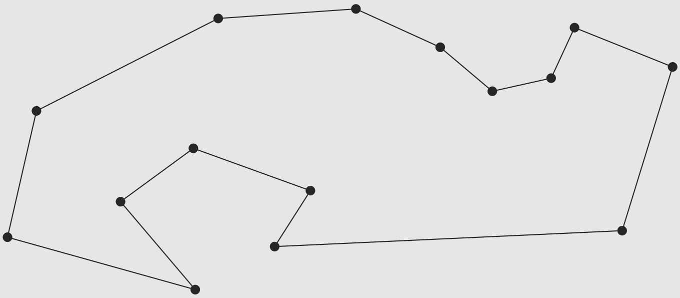
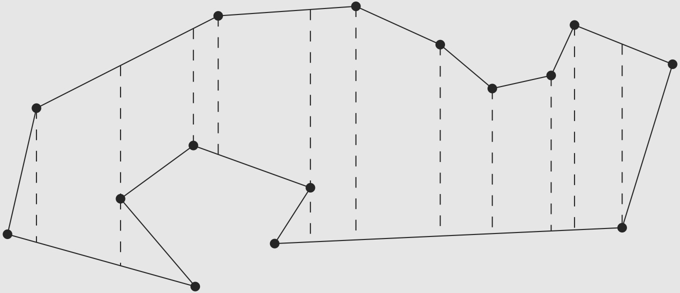
RI-Polygons

- All edges are RI



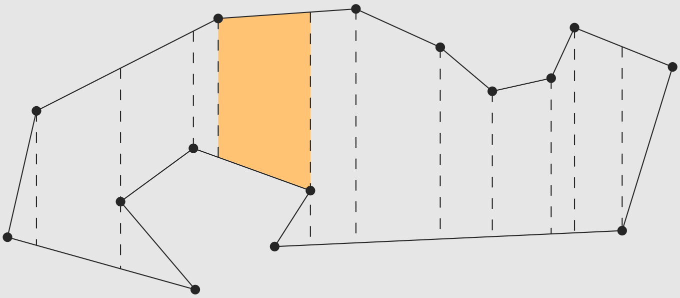
RI-Polygons

- Compute trapezoidal decomposition



RI-Polygons

- Add diagonal in alternating trapezoids



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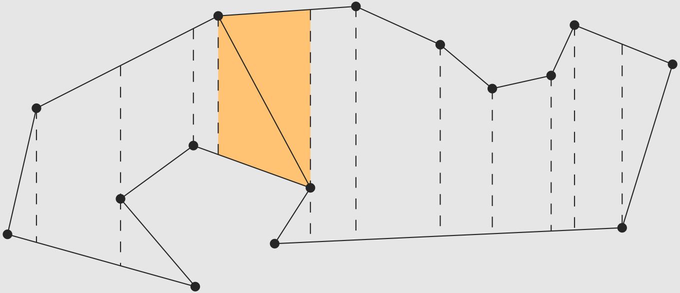

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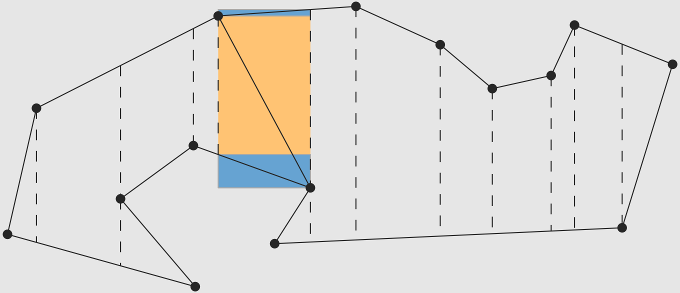

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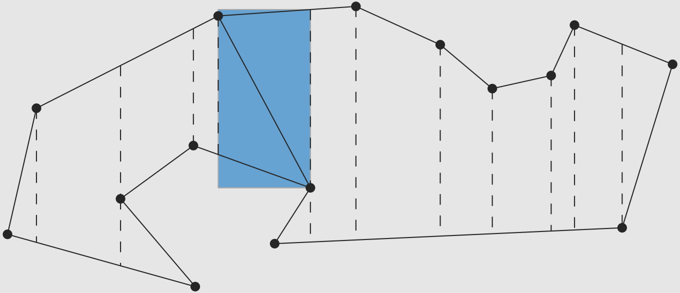

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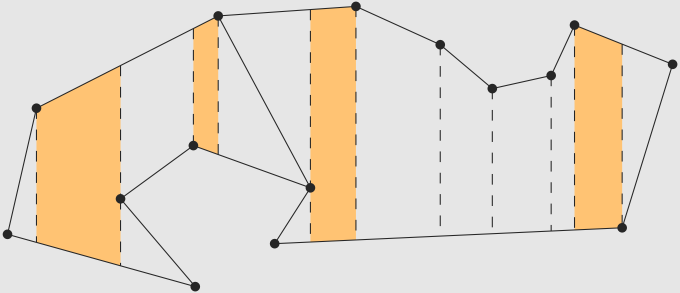

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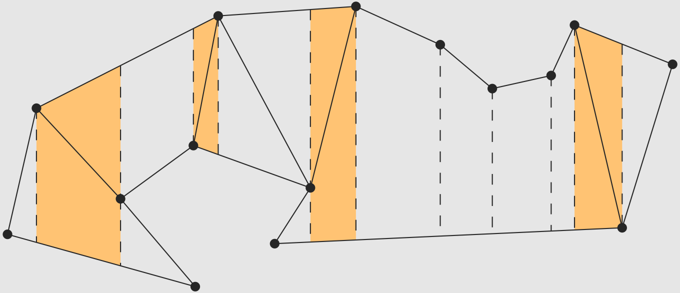

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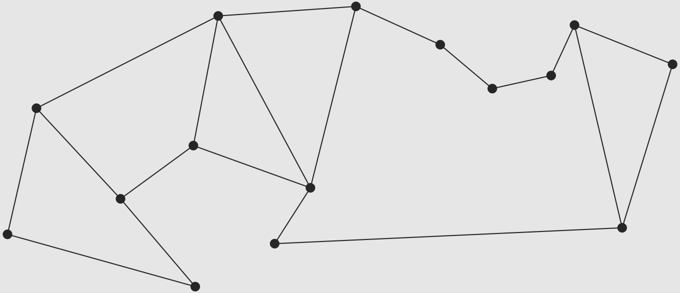
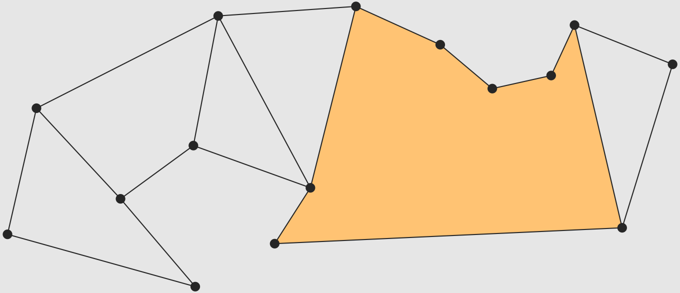


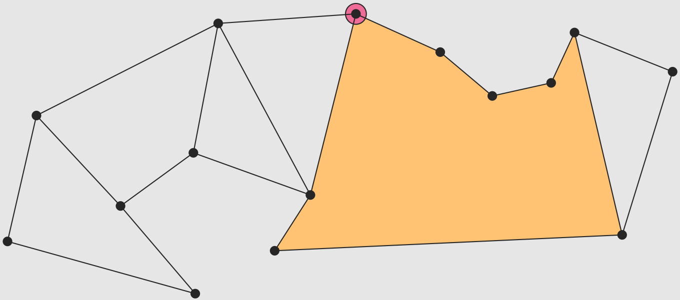
RI-Polygons

- Remaining pieces are *x*-monotone and one-sided



RI-Polygons

- Connect neighbours of local maximum



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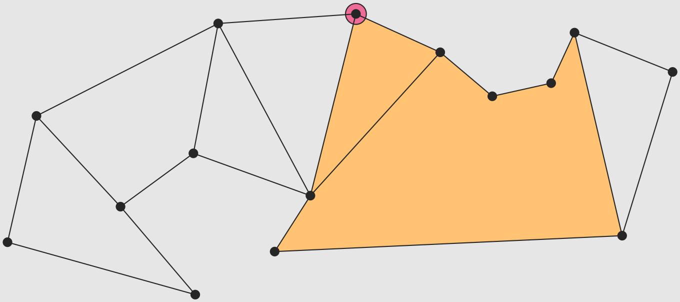

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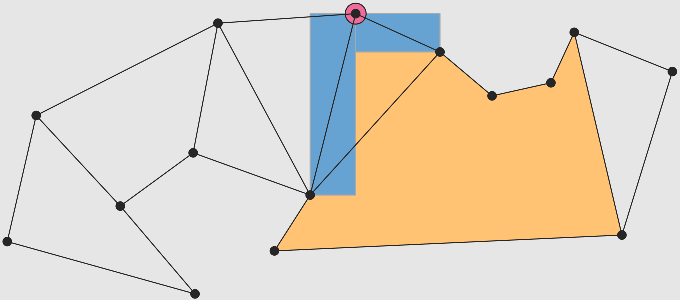

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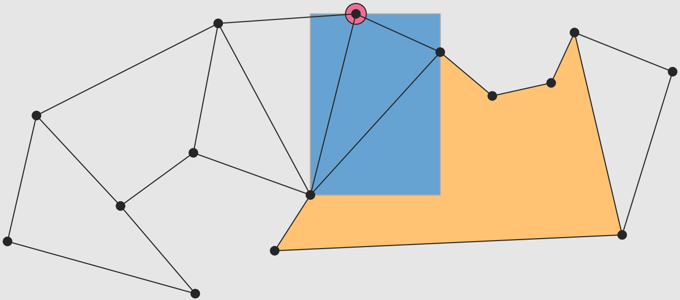

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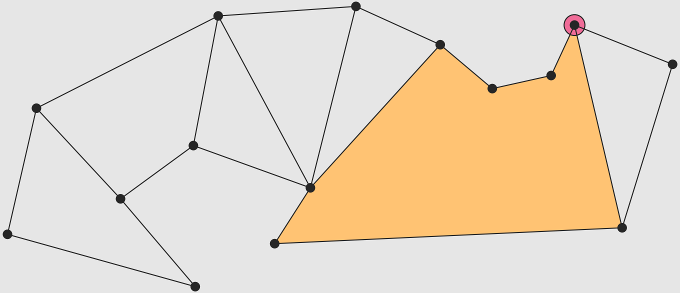

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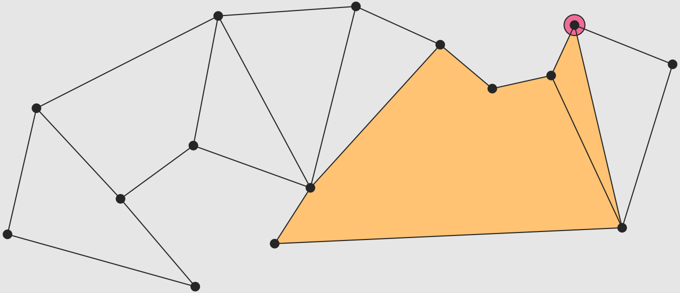

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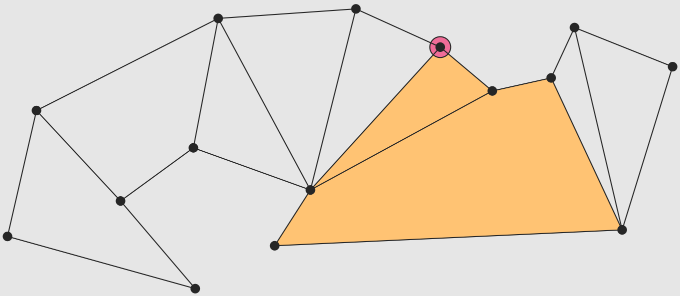

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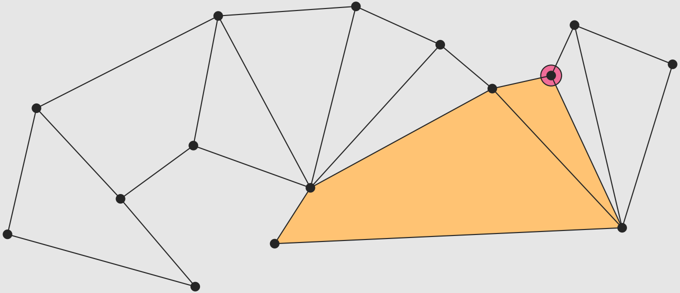

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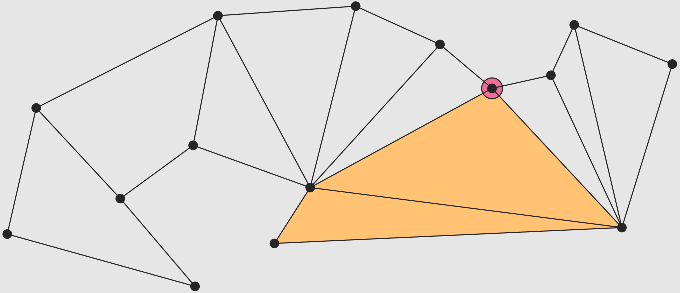

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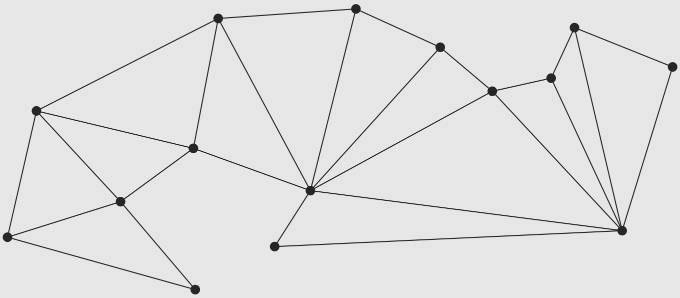
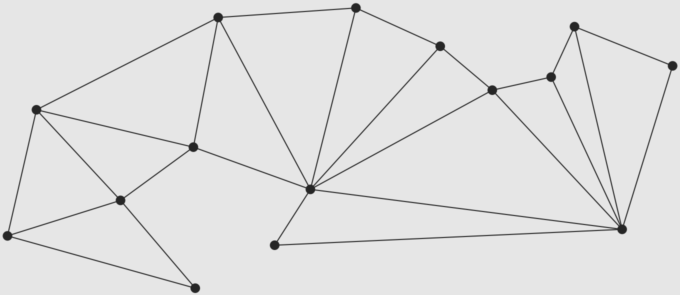


RI-Polygons

Theorem

Every RI-polygon can be RI-triangulated in linear time.

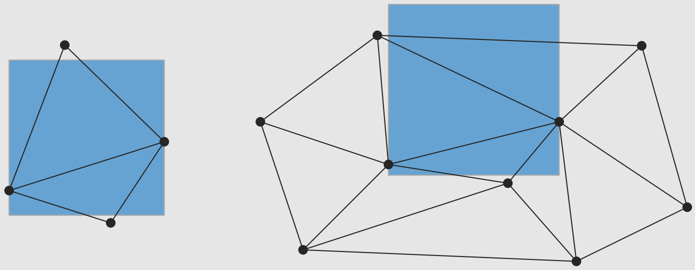


RI-Problems

1. RI-triangulating a polygon ✓
2. **RI-triangulating a point set**
3. Flipping one RI-triangulation to another
4. Flipping a triangulation to an RI-triangulation

RI-Point Sets

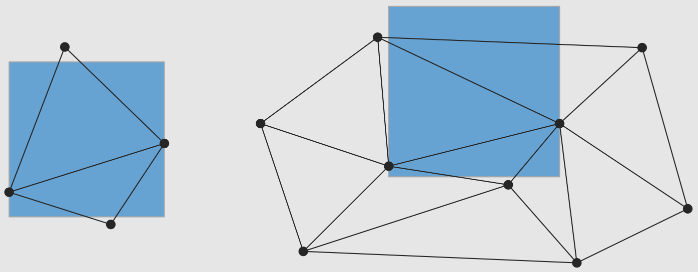
- The L_∞-Delaunay triangulation is RI



RI-Point Sets

Theorem

Any point set can be RI-triangulated in $O(n \log n)$ time.



RI-Problems

1. RI-triangulating a polygon ✓
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3. **Flipping one RI-triangulation to another**
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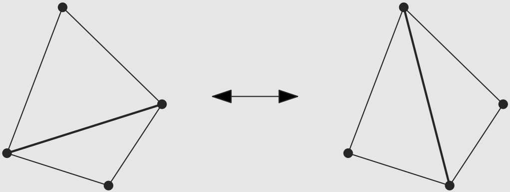
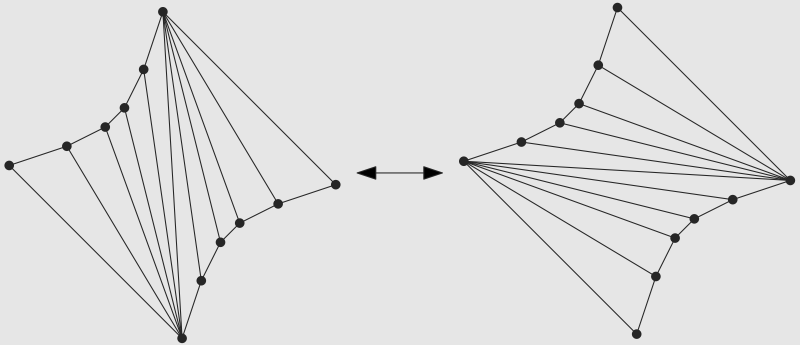
RI-Flips

- Exchange one diagonal of a convex quad for the other



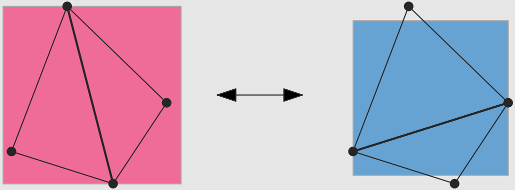
RI-Flips

- Is the class of RI-triangulations closed under flips?
- Diameter is $\Omega(n^2)$



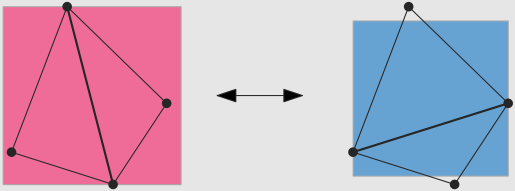
RI-Flips

- Transform into the L_∞-Delaunay triangulation
- An edge is locally L_∞ if its is L_∞ w.r.t. its neighbouring triangles
- If all edges are locally L_∞, we are in the L_∞-Delaunay triangulation



RI-Flips

- Flip edges that are not locally L_∞
- How do we know that new edge is (globally) RI?



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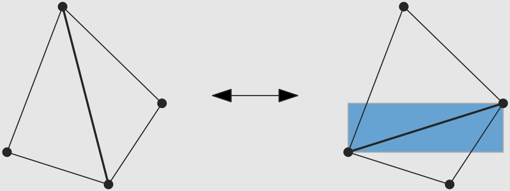

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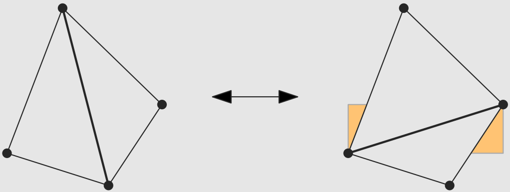

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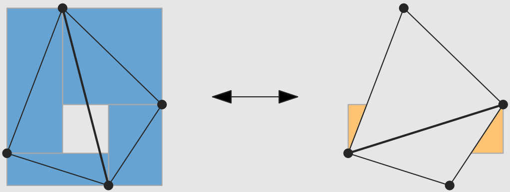

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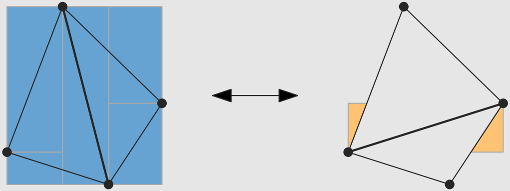

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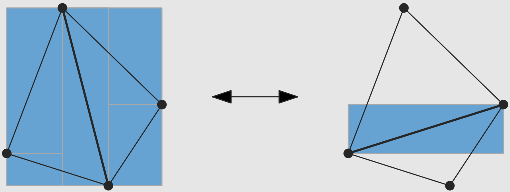
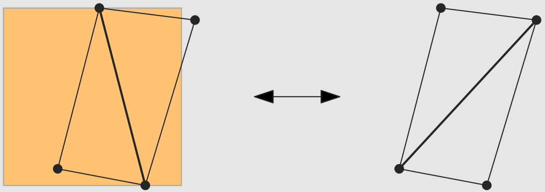


RI-Flips

- How many flips do we need?
- Give every edge a supporting square and count points inside



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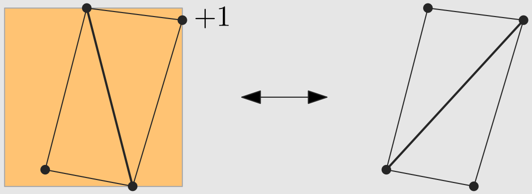

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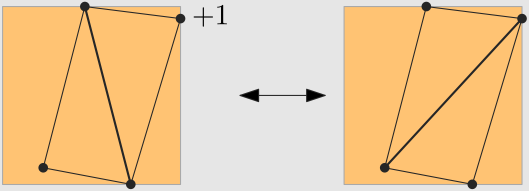

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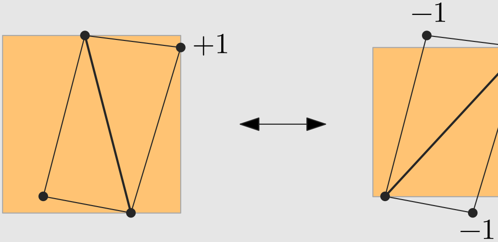
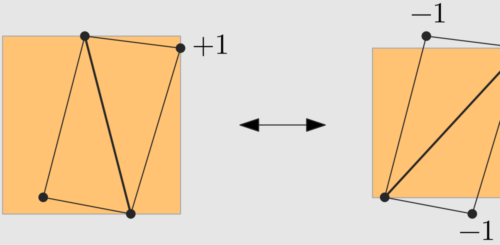


RI-Flips

Theorem

The class of RI-triangulations is closed under flips and its diameter is $\Theta(n^2)$.



RI-Problems

1. RI-triangulating a polygon ✓
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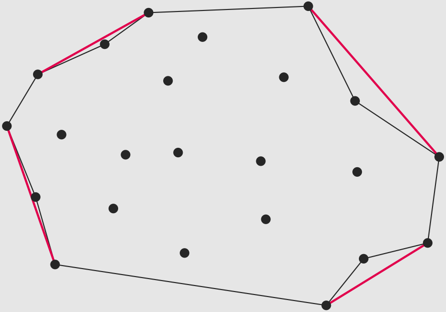
3. Flipping one RI-triangulation to another ✓
4. **Flipping a triangulation to an RI-triangulation**

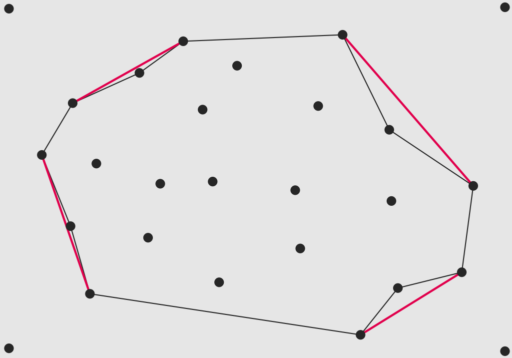
RI-Point Sets

- The outer face can be messy



RI-Point Sets

- The outer face can be messy
- We add 4 points ‘far away’ to deal with this



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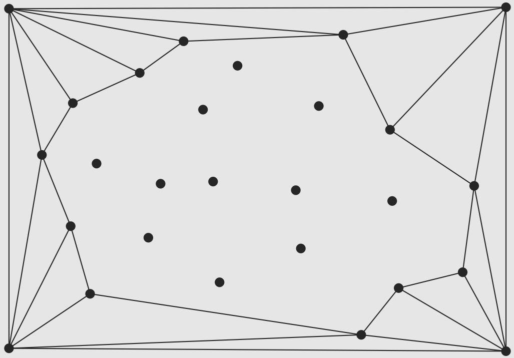
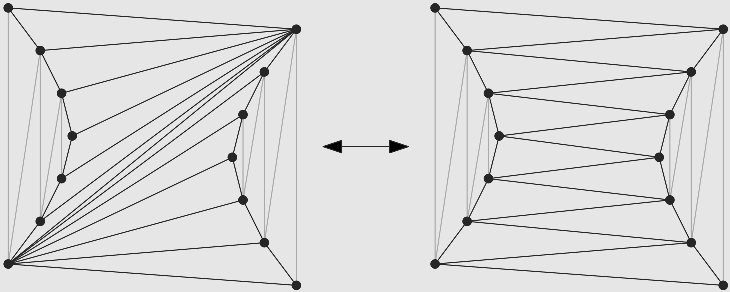


RI-Flips

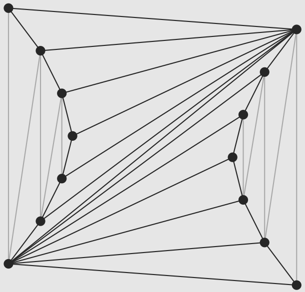
- Can we flip an arbitrary triangulation into an RI one?



RI-Flips

- Can we flip an arbitrary triangulation into an RI one?

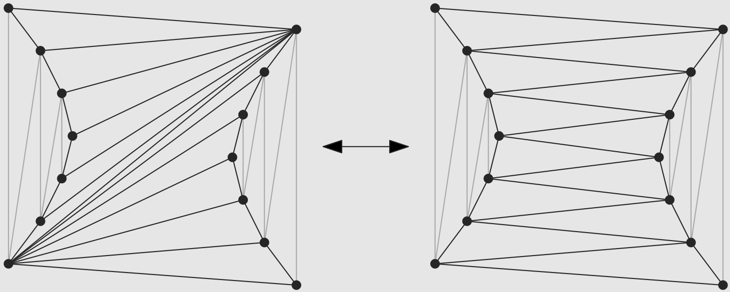
- Any triangulation can be flipped to any other in $O(n^2)$ flips [Lawson, 1972]



RI-Flips

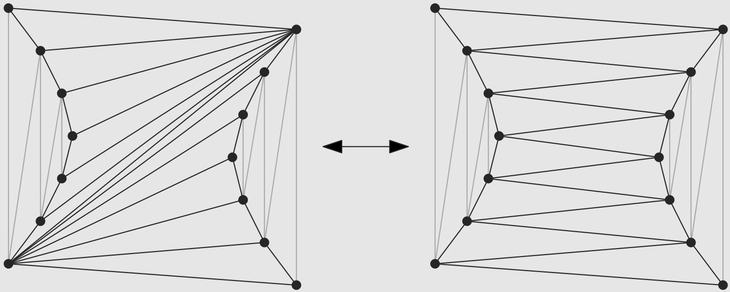
- Can we flip an arbitrary triangulation into an RI one?

- Any triangulation can be flipped to any other in $O(n^2)$ flips [Lawson, 1972]
- Some triangulations cannot be made RI in fewer than $\Omega(n^2)$ flips



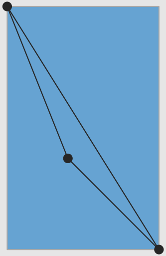
RI-Flips

- Can we flip an arbitrary triangulation into an RI one? **While getting monotonically 'closer'?**
- Any triangulation can be flipped to any other in $O(n^2)$ flips [Lawson, 1972]
- Some triangulations cannot be made RI in fewer than $\Omega(n^2)$ flips



RI-Flips

- Count points in ‘bad regions’
- No bad regions $\Rightarrow$ the triangulation is RI



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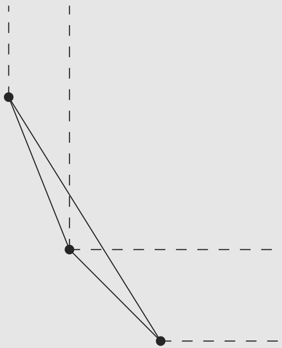

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- Count points in ‘bad regions’
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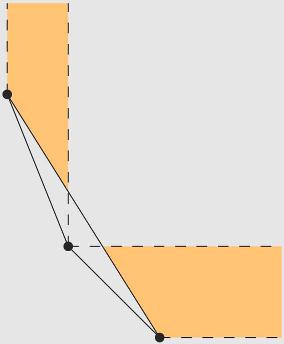
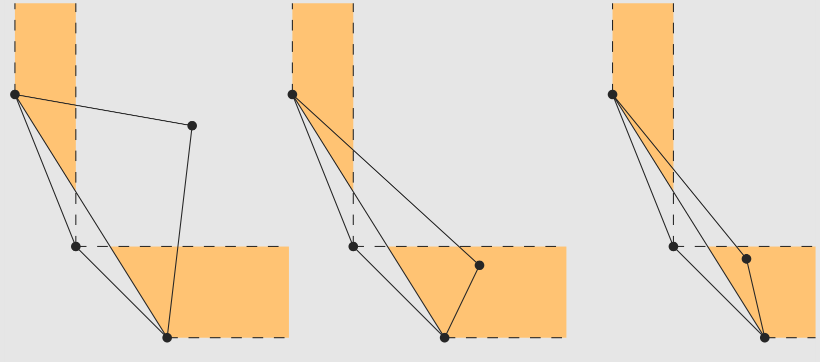


RI-Flips

- Flip the L_1-longest bad edge



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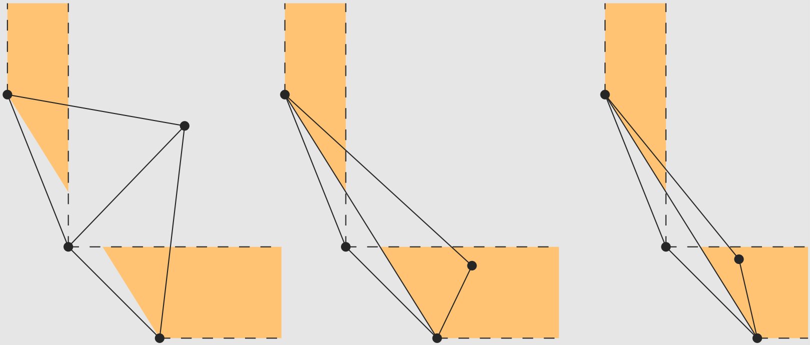

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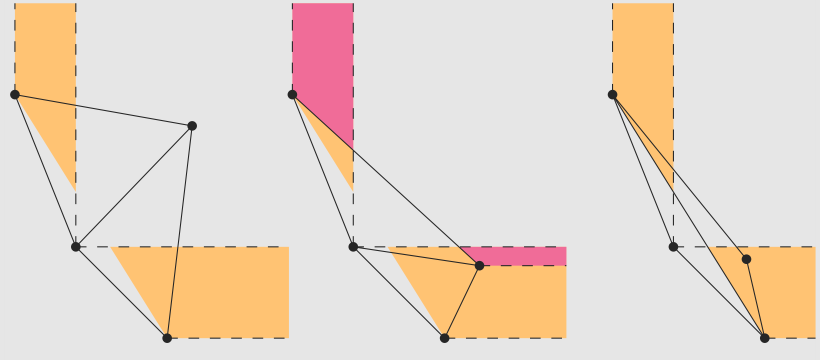

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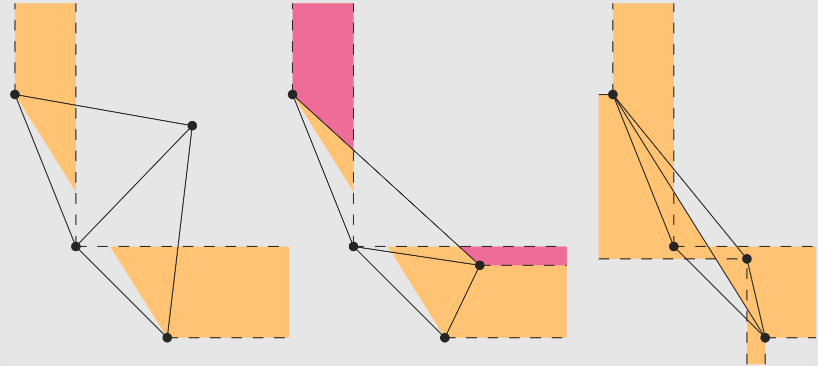

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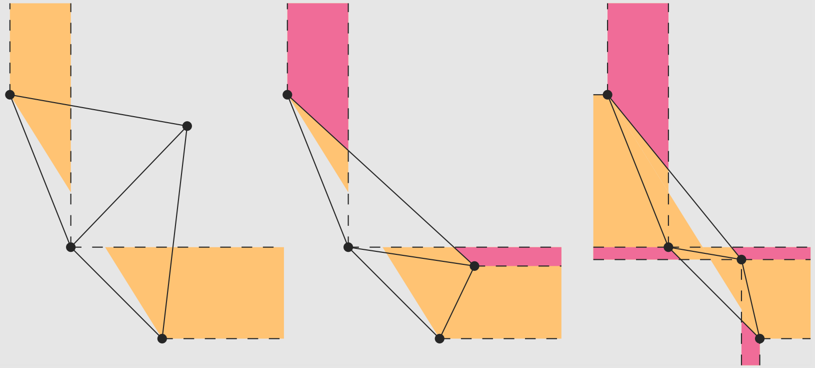
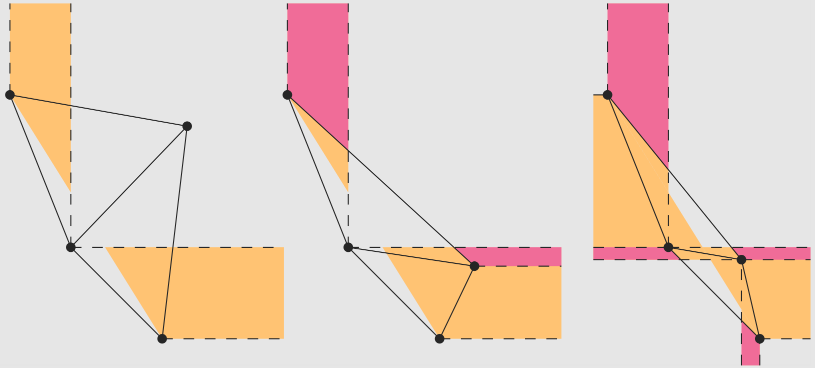


RI-Flips

Theorem

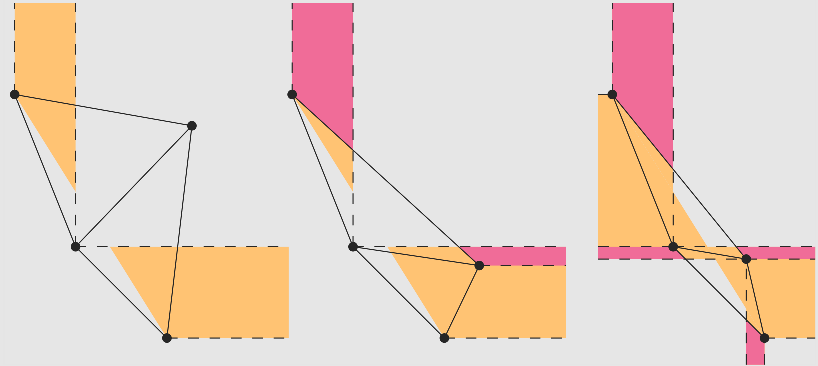
Any augmented triangulation can be converted into an RI-triangulation with $O(n^2)$ flips.



RI-Flips

Theorem

Any maximal triangulation can be converted into an RI-triangulation with $O(n^2)$ flips and $O(n)$ edge deletions.



RI-Problems

1. RI-triangulating a polygon ✓
2. RI-triangulating a point set ✓
3. Flipping one RI-triangulation to another ✓
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RI-Summary

- Any polygon or point set can be RI-triangulated
- Any two RI-triangulations can be transformed into each other with $\Theta(n^2)$ flips
- Any triangulation can be transformed monotonically into an RI-triangulation with $\Theta(n^2)$ flips