COMP 1805 Discrete Structures
Assignment 1

Due Tuesday, May 15th, 2012 at break (before 7:30pm)

Write down your name and student number on **every** page. The questions should be answered in order and your assignment sheets must be stapled, otherwise the assignment will not be marked. Total marks are 43.

1. (**4 marks**) For each of the following statements identify if the statement is a proposition, and if so what is its truth value?
   (a) $1 + 2 = 3$
   (b) Help me understand this logic stuff!
   (c) 7 is an even number.
   (d) $x + y = 4$
2. (**10 marks**) Prove or disprove the following: (you may use truth tables, or logical equivalences or any other valid form of argument).
   (a) $p \wedge q$ is equivalent to $\neg p \vee \neg q$
   (b) $\neg(p \vee \neg q)$ is equivalent to $\neg p \vee q$
   (c) $\neg(p \rightarrow \neg q)$ is equivalent to $p \wedge \neg q$
   (d) $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$
   (e) $p \oplus q$ is equivalent to $\neg p \oplus \neg q$
3. (**2 marks**) Determine whether or not the associative law holds for $\oplus$ (exclusive or).
4. (**6 marks**) Determine if the following are tautologies, contradictions or contingencies. You cannot use truth tables to justify your answers. Use either logical equivalences or some other means that does not use truth tables.
   (a) $((a \vee c) \wedge (b \vee c)) \vee ((c \rightarrow \neg b) \wedge (c \rightarrow a))$
   (b) $\neg((\neg a \rightarrow (\neg b \vee c)) \leftrightarrow (b \rightarrow (a \vee c)))$
   (c) $((a \vee b) \wedge (a \rightarrow c)) \rightarrow (b \vee c)$
5. (**2 marks**) Express the statement $a \oplus b$ using the other logical operators (eg. $\neg$, $\vee$, $\wedge$, etc.). In other words, write a logical statement that is equivalent to $a \oplus b$ without using $\oplus$.
6. (**4 marks**) Translate the following statements into English where $W(x)$ is "x has wings", $F(x)$ is "x can fly", and $Q(x)$ is "x quacks", and the domain is all animals.
   (a) $\exists x(W(x) \wedge \neg F(x))$.
   (b) $\forall x\,[(W(x) \wedge Q(x)) \rightarrow F(x)]$.
7. (**5 marks**) Translate the following English statements into propositional logic. Define the propositions you will use. Your base propositions should not be negative or compound propositions themselves. Use brackets when necessary to make the order of evaluation clear.
   (a) If I study Discrete Math and Linear Algebra then I will get good grades.
   (b) The sky is blue or the earth is round.

(c) It is Monday and I am not at work.

(d) I will live to be 100 years old iff I excercise every day and I eat well.

(e) I miss the bus if I sleep in.

8. (**2 marks**) Give the negation of the following statement. The negation symbols should proceed the predicates. $\exists x \forall y \exists z \left[A(x, y) \rightarrow (B(x, y) \vee C(x, y))\right]$

9. (**6 marks**) Let $H(x)$ be "x plays hockey", $S(x)$ be "x can skate", $D(x)$ be "x can dance" and $E(x)$ be "x earns money". The universe of discourse is all humans. State the following logically.

   (a) Everyone that can skate and dance earns money.

   (b) At most two people can play hockey and dance.

   (c) Not everyone who plays hockey earns money.

10. (**2 marks**) Let $L(x)$ be "x is a Lion" and $M(x)$ be "x eats meat.", where the universe of discourse is all animals. Are the statements "$\forall x(L(x) \rightarrow M(x))$" and "$\forall x(L(x) \wedge M(x))$" logically the same thing? Prove your answer.