

# COMP 1805 Discrete Structures

## Assignment 2

Due Thursday, May 24<sup>th</sup>, 2012 at break

Write down your name and student number on **every** page. The questions should be answered in order and your assignment sheets must be stapled, otherwise the assignment will not be marked. Total marks are 37.

1. (3 marks) Use rules of inference to determine if "I will feel tired" is a valid conclusion from the following premises "If it is the weekend I will go shopping", "It is the weekend or I will go out to dinner", "If I go out to dinner then I will eat too much", "If I eat too much I will feel tired", and "I will not go shopping".
2. (3 marks) Use rules of inference for quantifiers to show that the statement "Andrew can count by twos and tie his shoes" is true for Andrew the university student. The premises are "All university students that have finished high school can count to fifty and tie their shoes", "All university students have finished highschool and can count by twos".
3. (3 marks) Prove or disprove that  $1 + 3\sqrt{2}$  is irrational.
4. (2 marks) Prove or disprove the following: If  $x$  and  $y$  are rational, then  $x^y$  is rational.
5. (2 marks) Prove or disprove that if set  $A$  and set  $B$  both have the same cardinality, then the power set of  $A$  is identical to the power set of  $B$ .
6. (6 marks) Determine whether or not the following are valid. Justify your answer by using set identities or membership tables as indicated. Let  $A, B$  and  $C$  be sets.
  - (a)  $(A \oplus B) = (A - B) \cup (B - A)$  (membership table).
  - (b)  $(A - B) \cup (B - A) = \overline{(A \cap B)}$  (membership table).
  - (c)  $(C - A) \cap (B - A) = (C \cap B) \cap \overline{A}$  (set identities).
7. (6 marks) Draw the Venn Diagram of the following:
  - (a)  $(A \oplus B)$
  - (b)  $\overline{(B \cap C)} \cup \overline{A}$
  - (c)  $(B \cap C) \cup \overline{A \cap B}$
8. (2 marks) Prove or disprove that if set  $A$  and set  $B$  both have the same power set, then  $A = B$ .
9. (8 marks) Indicate if each of the following statements is true or false. As was done in class you should provide witnesses to demonstrate when a function is of the order specified.
  - (a)  $3n^4 + 107n^3 + 5$  is  $O(n^5)$
  - (b)  $17n^2 + 23n + 2$  is  $O(n^4)$
  - (c)  $\frac{1}{4}n \log n$  is  $O(n)$
  - (d)  $45 + 1/n$  is  $\Theta(1)$
10. (2 marks) If  $f(x)$  is  $\Omega(g(x))$  and  $f(x)$  is  $O(h(x))$  then is it true that  $g(x)$  is  $O(h(x))$ ?