COMP 1805 Discrete Structures
Assignment 3

Due Thursday, June 7th 2012 at the break

Write down your name and student number on **every** page. The questions should be answered in order and your assignment sheets must be stapled, otherwise the assignment will not be marked. Total marks are 51.

1. (**4 marks**) Compute the exact sums of the following summations. You must show your work so you will not be awarded any marks for simply writing down a number for your answer - regardless of whether it is right or wrong.

   (a) $\sum_{k=1}^{n}(8k + 4 + 12k^2)$

   (b) $\sum_{j=13}^{29} 6j$

   (c) $\sum_{i=1}^{6} 4 \cdot 2^{i-1}$

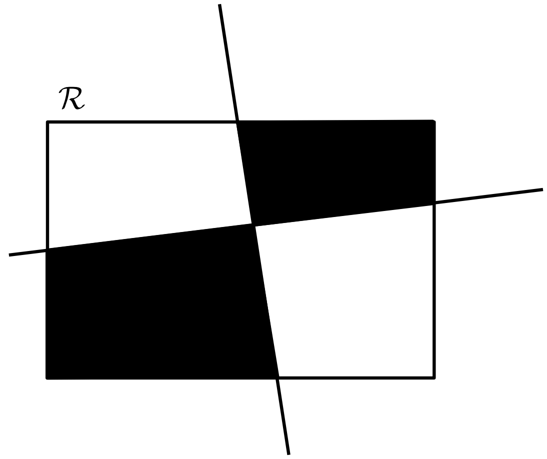
   (d) $\sum_{x=0}^{10}\sum_{y=0}^{3}(xy + x + y + 1)$

2. (**2 marks**) A wheel of fortune has the integers from 1 to 25 placed on it in a random fashion. Show that no matter how the numbers are arranged on the wheel there will always be at least one grouping of three consecutive numbers which sum to at least 39.

3. (**4 marks**) Use mathematical induction to show that $\sum_{j=0}^{n}(j+1) = \frac{(n+1)(n+2)}{2}$.

4. (**4 marks**) Assume we have a rectangle, $\mathcal{R}$ through which we can draw any number of lines. The first line we draw (we assume any line we draw intersects $\mathcal{R}$) subdivides $\mathcal{R}$ into two *faces*. Any subsequent line we draw will likewise split one or more of the existing faces, and create a new face(s). Any face we intersect is split into exactly two new faces. We say the lines meet at a *vertex* and that portion of a line between two *vertices* is an *edge*. Two faces are *adjacent* if and only if they share an edge. Now assume we want to colour the faces in $\mathcal{R}$ black or white. Prove using induction that it is always possible to colour the faces in $\mathcal{R}$ such that no two adjacent faces are coloured the same. Figure 4 below shows such a subdivision and colouring with one and two lines.

(a)



(b)

5. (**4 marks**) Use mathematical induction to prove that $n! \leq n^n$.
6. (**4 marks**) Assume that you live in a country where the only currency are 3$ and $10 dollar bills. Show that every amount greater than $17 can be made from these bills.

   Let $P(n)$ be the statement that a postage of n cents can be formed using just 4 and 7 cent stamps. Use *strong induction* to prove that $P(n)$ is true for $n \geq 18$.
7. (**3 marks**) Provide a recursive definition for the following sequences where a_n, $n = 1, 2, 3, \ldots$:
   (a) $a_n = 4$
   (b) $a_n = \frac{n}{2} + 3$
   (c) $a_n = n!$
8. (**2 marks**) How many bits strings are there of length less than ten and with an odd number of bits?
9. (**2 marks**) How many numbers between 0 (000,000 use 0 as a place holder for numbers less than 100,000) and 999,999 contain at least two even digits. Assume that placeholder 0's count as even digits (eg 111,123 has one even digit and 001,234 has 4 even digits).
10. (**2 marks**) How many integers greater than 100 and less than 1000 are divisibile by 3, 4, or 7?
11. (**2 marks**) How many non-negative integers greater than 313 but less than 2029 are divisible by 13.
12. (**10 marks**) Assume we have an alphabet with just the characters $\{\alpha, \epsilon, \gamma, \tau, \omega, \beta, \sigma, \pi\}$. Answer the following questions about this alphabet.
    (a) Allowing repitition, how many words of length 7 start with $\alpha\beta$ or end with σ?
    (b) Allowing repitition how many words can we form of length 4 that contain the sequence $\pi\pi$?
    (c) How many words of length 6 can be formed where each character is unique?
    (d) How many permutations of the letters contain the string $\beta\pi$?
    (e) How many permutations contain the strings $\alpha\beta\gamma$ and $\epsilon\tau\pi$?
13. (**6 marks**) Say we have a group of 10 dogs, 12 cats, and 6 chickens:
    (a) How many different groups of 7 animals can we make.
    (b) How many different groups of 5 animals have at least 1 cat.
    (c) How many different groups of 5 animals have at least one dog, one cat if we can only pick dogs and cats.
14. (**2 marks**) Let n be an integer where $n > 2$. How many bit strings of length n, *or less*, are there that start with 1 and end with 1.