COMP 1805 Discrete Structures
Assignment 4

Due Thursday, June 14th, 2012 at break (before 7:30pm)

Write down your name and student number on **every** page. The questions should be answered in order and your assignment sheets must be stapled, otherwise the assignment will not be marked. Total marks are 30.

1. (**2 marks**) Let S be a 2 cm by 2 cm square. Prove that it is impossible to place five points inside S such that all points are more than $\sqrt{2}$ cm apart (Hint: use the pigeonhole principle).

2. (**2 marks**) How many solutions are there to the inequality

$$2x_1 + 2x_2 + 2x_3 < 32$$

   where x_1, x_2, x_3 are non-negative integers?

3. (**2 marks**) There is a video on Youtube of the muppets singing the song MAHNAMAHNA (the spaces have been dropped). How many different unique strings can be formed by rearranging the letters in the title of this song?

4. (**2 marks**) Assume a person is standing at point (3,5) on a discrete grid (all grid coordinates are integers) and assume that they can take steps in either the positive x or y directions (i.e. if they are at point (4,7) they can go to (5,7) or (4,8)). Using this rule, how many different routes are there from (3,5) to position (9,10). Note that two routes are considered different is they differ in any step.

5. (**6 marks**) Consider a relation R defined on the integers. Determine for the following if the relations are reflexive, symmetric, anti-symmetric, transitive.

   (a) $R = \{(a, b)|a = \frac{b}{2}\}$
   (b) $R = \{(a, b)|ab \geq 0\}$
   (c) $R = \{(a, b)|a > b^2\}$

6. (**2 marks**) Let A be a set of size 4. How many reflexive relations on A are there?

7. (**4 marks**) Use Warshall's algorithm to find the transitive closures of the following relations on $\{a, b, c, d, e\}$. By use Warshall's algorithm it is meant that you should draw the state of the matrices W_0, W_1, etc.

   (a) $\{(a, d), (b, c), (c, b), (c, d), (d, c), (e, a), (e, e)\}$
   (b) $\{(a, a), (a, c), (b, e), (c, b), (d, e), (e, c)\}$

8. (**2 marks**) Let R be a reflexive relation defined on the set D. Prove or disprove that R^n is a reflexive relation for all $n \geq 1$.

9. (**2 marks**) Show that the relation consisting of all (x, y) such that x and y are bit strings of length 5 or more that agree except perhaps in their first five bits is an equivalence relation on the set of all bit strings of length five or more.

10. (**6 marks**) Consider the Hasse diagram shown below on the set $\{a, b, c, d, e, f, g, h, i, j, k, l\}$. Answer the following questions (when listing elements of a solution set please list them in alphabetical order if you can do so).

    (a) What are the maximal elements.

(b) What are the minimal elements.

(c) Is there a greatest element.

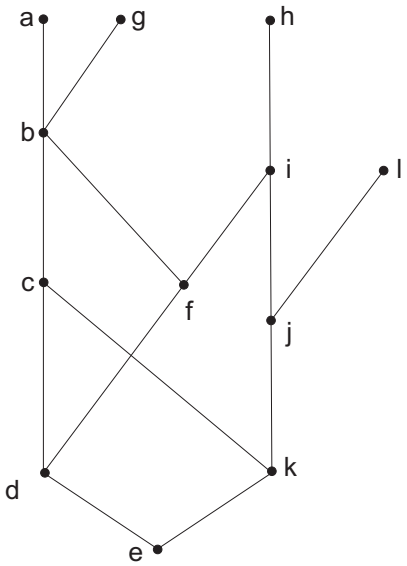
(d) Is there a least element.

(e) Find all upper bounds for $\{d, e, k\}$.

(f) Find all lower bounds for $\{a, g, h\}$.

(g) Find the greatest lower bound for $\{a, g, h\}$ if it exists.

(h) Use topological sort (showing your steps as in class and in the textbook in Figures 10 and 11 of Chapter 8) to generate a compatible total order on the set.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