Carleton University

Final
Examination
Summer 2012

DURATION: 3 HOURS

No. of students: 55

Department Name & Course Number: **Computer Science COMP/MATH 1805A**
Course Instructor: Craig Dillabaugh

Authorized memoranda:
Non-programmable calculator

Students MUST count the number of pages in this examination question paper before beginning to write, and report any discrepancy to the proctor. This question paper has 15 pages (not including the cover page).

This examination question paper MAY NOT be taken from the examination room.

In addition to this question paper, students require:

an examination booklet:	**no**
a Scantron sheet:	**yes**

Student Name:

Student Number:

Instructions:

1. This is a closed book exam. No notes or other aids are allowed.
2. All questions must be answered on this examination paper as well as on the scantron sheet.

Marking scheme: Each question is worth 2 marks.

1. (**2 marks**) Let $C(x, y)$ be a predicate that depends on x and y. What is the negation of $\forall x \exists y : C(x, y)$?

   (a) $\exists x \exists y : C(x, y)$
   (b) $\exists x \forall y : \neg C(x, y)$
   (c) $\exists x \forall y : C(x, y)$
   (d) $\exists x \exists y : \neg C(x, y)$

   Solution:
   Note that $\neg \forall x \exists y : C(x, y) \equiv \exists x \neg \exists y : C(x, y) \equiv \exists x \forall y : \neg C(x, y)$ so (b).

2. (**2 marks**) Let $A = \{x, y, 7, \{7, 5\}, \{6, 7, 3, 3, 2\}, u, v, \{3\}, \{7\}\}$. The number of elements in the power set of A is

   (a) 9
   (b) 13
   (c) 2^{13}
   (d) 2^9

   Solution:
   The set contains 9 unique elements as as such has 2^9 elements in its power set, so (d) is the correct answer.

3. (**2 marks**) How many solutions are there to the equation $x_1 + x_2 + x_3 = 16$, where $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ are integers?

   (a) $\binom{18}{2}$
   (b) $\binom{19}{2}$
   (c) $\binom{19}{3}$
   (d) $\binom{18}{3}$

   Solution:
   The correct answer here is a.

4. (**2 marks**) Let A be a set of size 6 and let B be a set of size 5. How many different functions are there from A to B that are **not** one-to-one?

   (a) 6^5
   (b) $5^6 - 5!$
   (c) 5^6
   (d) $6^5 - \binom{6}{5}$

   Solution:
   You are mapping from a set of size 6 to a set of size 5 so all functions are not one-to-one, so we just have to count the functions. For each of the 6 elements of the domain there are 5 possible choices in the codomain, so we get 5^6 possible functions or c.

5. (**2 marks**) Let A be a set of size 5. How many relations on A are there?

   (a) 2^{25}
   (b) $5!$
   (c) 2^{10}
   (d) $25!$

   Solution:
   a or 2^{n^2} with $n = 5$.

6. (**2 marks**) How many positive integers less than 1000 are divisible by NEITHER 7 NOR 11?

   (a) 119
   (b) 779
   (c) 889
   (d) 449

   Solution:
   Answer is b or 779 since there are 999 positive integers less than 1000 and so we have $999 - (\lfloor 1000/7 \rfloor + \lfloor 1000/11 \rfloor - \lfloor 1000/(11 * 7) \rfloor) = 799$.

7. (**2 marks**) Consider the following two statements:

$$\begin{aligned} P: &\quad \sum_{i=1}^{n}(7i^3 - 12i + 77) = O(n^4) \\ Q: &\quad \log(n!) = \Theta(n \log n) \end{aligned}$$

Which of the following is true?

(a) P is TRUE and Q is TRUE.
(b) P is TRUE and Q is FALSE.
(c) P is FALSE and Q is TRUE.
(d) P is FALSE and Q is FALSE.

Solution:
Dropping this one, its too hard, esp. the proving the lower bound on Q. However, if you are curious the correct answer is a