Counting Graphs

Some times we may want to enumerate certain properties or attributes of a graph. For example, how many edges can thier be in a directed graph (no self loops and not a multigraph), if the graph has n vertices.

Well there are n vertices. then each vertex can have an edge from itself, to any of the other vertices. So we can have $n \cdot n-1$ vertices (since the are no "self-loops") $= n^2 - n$

Think of the matrix represen tation

$$\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}$$

no loops - so there are no 'ones' here.

The matrix has n^2 elements of which n are never set (self-loops). So we have $n^2 - n$ possible edges.

Ex: How many directed graphs are there on a graph with n vertices?

Think of the matrix rep. Each bit can be 0 or 1 (edge absent / edge present). so there are as many simple directed graphs on a graph with n vertices as there are bit strings of length $n \cdot n-1 = n^2 - n$ which is $2^{(n^2-n)}$.

Note that this is the same as the number of reflexive relations on a set with n elements.

Ex: How many edges can there be in an undirected graph with n vertices.

Well, there are n vertices, an edge in the undirected graph, is represented by a pair of of vertices $\{a, b\}$. So there are as many edges as there are ways to pick a pair of vertices from among n vertices, which is $\binom{n}{2}$.