Section 4.3

1. Find the prime factorization for each of the following integers.
   (a) 88
   (b) 729
   (c) 1001
   (d) 909090
2. Which positive integers smaller than 30 are co-prime with 30 ?
3. With the help of a computer, calculate the number of prime numbers smaller than 2020.
4. For each pair of integers, detemine the greatest common divisor.
   (a) $a = 3^7 \cdot 5^3 \cdot 7^3$
   $b = 2^{11} \cdot 3^5 \cdot 5^9$
   (b) $a = 11 \cdot 13 \cdot 17$
   $b = 2^9 \cdot 3^7 \cdot 5^5 \cdot 7^3$
   (c) $a = 23^{31}$
   $b = 23^{17}$
   (d) $a = 41 \cdot 43 \cdot 53$
   $b = 41 \cdot 43 \cdot 53$
   (e) $a = 3^{13} \cdot 5^{17}$
   $b = 2^{12} \cdot 7^{21}$
5. If the product of two integers is $2^7 \cdot 3^8 \cdot 5^2 \cdot 7^{11}$ and their greatest common divisor is $2^3 \cdot 3^4 \cdot 5$, what is their least common multiple ?
6. Use Euclid's algorithm to find
   (a) $\gcd(1, 5)$
   (b) $\gcd(100, 101)$
   (c) $\gcd(123, 277)$
   (d) $\gcd(1529, 14039)$
   (e) $\gcd(1529, 14038)$
   (f) $\gcd(11111, 11111)$
7. Find two integers s and t such that
   (a) $3s + 4t = 1$
   (b) $1534s + 2020t = 1608$
   (c) $25s + 36t = 3$
   (d) $17s + 31t = 1$

For each of the following statements,

— Write the statement using quantifiers and logical operators.

— Determine if the statement is true or false.

— Show that your answer is correct.

— Indicate the method of proof used.

8. Let $n > 1$ be an integer. If n is not prime, then n has a prime divisor p such that $p \leq \sqrt{n}$.
9. Let $n > 1$ be an integer. If n has a prime divisor p such that $p \leq \sqrt{n}$, then n is not prime.
10. Let a, b, q, r be integers such that $\gcd(a, b) = \gcd(b, r)$. Then $a = bq + r$.
11. Let a, b, q, r be integers such that $a = bq + r$. Then $\gcd(a, b) = \gcd(b, r)$.
12. Let $a, b, c \in \mathbb{Z}$ with $a > 0$, $b > 0$ and $c > 0$. If $\gcd(a, bc) = 1$, then $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$.
13. Let $a, b, c \in \mathbb{Z}$ be positive integers. If $\gcd(a, b) = 1$ and $a|bc$, then $a|c$.
14. Let $a, b \in \mathbb{Z}$ be positive integers. If $\gcd(a, b) = d$, then $\gcd(a^2, b^2) = d^2$.
15. Let $a, b, c \in \mathbb{Z}$ be positive integers. If $\gcd(a, b) = 1$ and $a|c$, then $a|bc$.
16. Let $a, b, c \in \mathbb{Z}$ and $m \geq 2$ be integers. If $a \equiv b \pmod{m}$ and $a \cdot c \equiv b \cdot c \pmod{m}$, then $\gcd(c, m) = 1$.