

Graphs

- Let $G = (V, E)$ be a non-directed graph. Set $v = |V|$ and $e = |E|$. True or false? Prove or disprove the following statements.
 - If $e \leq 3v - 6$, then G is planar.
 - If G is planar, then $e \leq 3v - 6$.
 - If G is planar, then V contains at least one vertex of degree at most 5.
 - If V contains at least one vertex of degree at most 5, then G is planar.
- Draw K_n for $1 \leq k \leq 10$.
- Draw C_n for $3 \leq k \leq 10$.
- Draw $K_{m,n}$ for $1 \leq m \leq n \leq 6$.
- We demonstrated in class that all planar graphs are 6-colorable. The proof leads directly to an algorithm to 6-color a planar graph. Describe this algorithm.
- Apply the algorithm from question 5 to the following graph (see figure 1).

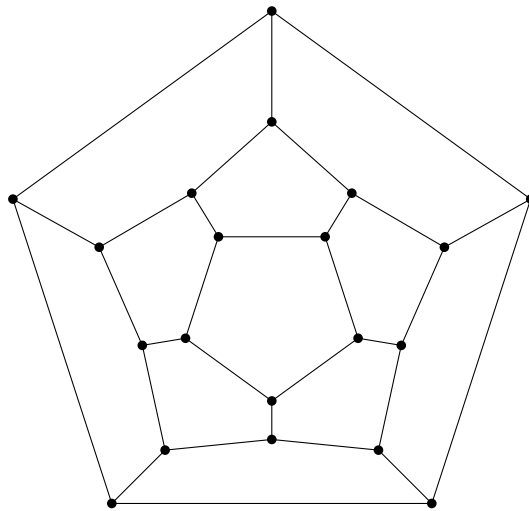


FIGURE 1 – Figure for question 6

- Apply the algorithm from question 5 to the following graph (see figure 2).
- For what n values is K_n planar? Prove that your answer is correct.
- For what n values is C_n planar? Prove that your answer is correct.
- For what m and n values is $K_{m,n}$ planar? Prove that your answer is correct.
- Draw all possible graphs with at most 5 vertices.
 - Draw all possible planar graphs with at most 5 vertices.

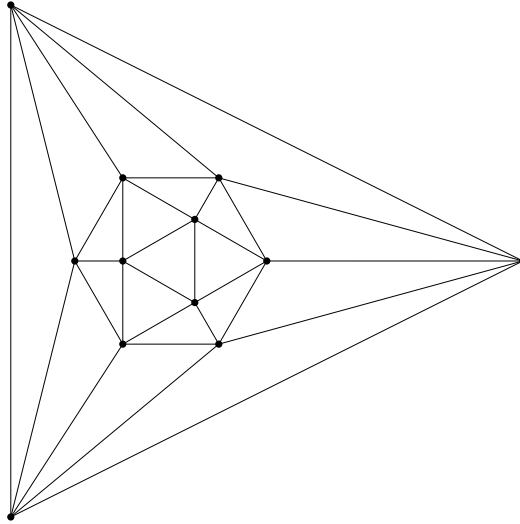


FIGURE 2 – Figure for question 7

- (c) Draw all possible connected graphs with at most 5 vertices.
- (d) Draw all possible connected planar graphs with at most 5 vertices.
12. Among all the graphs drawn in question 11, which are bipartite?
 13. For all $n \geq 3$, find the chromatic number for C_n .
 14. For all $n \geq 1$, find the chromatic number for K_n .
 15. For all $1 \leq m \leq n$, find the chromatic number for $K_{m,n}$.
 16. For what n values does K_n contain an Eulerian cycle? Prove that your answer is correct.
 17. For what n values does C_n contain an Eulerian cycle? Prove that your answer is correct.
 18. For what m and n values does $K_{m,n}$ contain an Eulerian cycle? Prove that your answer is correct.
 19. For what n values does K_n contain an Eulerian path (that is not a cycle)? Prove that your answer is correct.
 20. For what n values does C_n contain an Eulerian path (that is not a cycle)? Prove that your answer is correct.
 21. For what m and n values does $K_{m,n}$ contain an Eulerian path (that is not a cycle)? Prove that your answer is correct.
 22. For what n values does K_n contain a Hamiltonian cycle? Prove that your answer is correct.
 23. For what n values does C_n contain a Hamiltonian cycle? Prove that your answer is correct.
 24. For what m and n values does $K_{m,n}$ contain a Hamiltonian cycle? Prove that your answer is correct.

25. For what n values does K_n contain a Hamiltonian path (that is not a cycle)? Prove that your answer is correct.
26. For what n values does C_n contain a Hamiltonian path (that is not a cycle)? Prove that your answer is correct.
27. For what m and n values does $K_{m,n}$ contain a Hamiltonian path (that is not a cycle)? Prove that your answer is correct.
28. For all $n \geq 3$, find the size of a maximum matching in C_n .
29. For all $n \geq 1$, find the size of a maximum matching in K_n .
30. For all $1 \leq m \leq n$, find the size of a maximum matching in $K_{m,n}$.