COMP 3803 — Assignment 1

Due: Wednesday October 5, 23:59.

Assignment Policy:

- Your assignment must be submitted as one single PDF file through Brightspace.

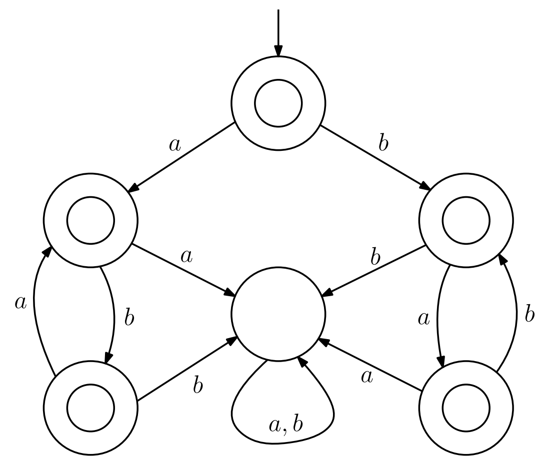
> Use the following format to name your file:
>
> `LastName_StudentId_a1.pdf`

- **Late assignments will not be accepted. I will not reply to emails of the type "my internet connection broke down at 23:57" or "my scanner stopped working at 23:58", or "my dog ate my laptop charger".**
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

> When specifying a finite automaton, it is sufficient to draw the state diagram (because this diagram tells us what are the alphabet, the set of states, the start state, the set of accept states, and the transition function). However, you must explain the meaning of the different states that you use.

Question 1: Write your name and student number.

Question 2: What is the language of the following DFA? The alphabet is $\{a, b\}$. Justify your answer.



Question 3: For each of the following two languages, construct a DFA that accepts the language. In both cases, the alphabet is $\{a, b\}$. For each DFA, justify correctness.
(3.1) The language consisting of all strings $w \in \{a, b\}^*$ that start and end with b.
Note that the string b (having length one) is included in this language.
(3.2) The language consisting of all strings $w \in \{a, b\}^*$ in which the number of a's is even and the number of b's is a multiple of three.
Note that the empty string ε is included in this language.

Question 4: Construct an NFA with four states whose language is the set of all strings $w \in \{a, b\}^*$ such that

- $w = a^k$, for some integer $k \geq 0$, or
- $w = (ab)^k$, for some integer $k \geq 0$.

As always, justify correctness.

Question 5: Construct an NFA whose language is the set of all strings $w \in \{a, b\}^*$ such that

- $w = (aab)^k a$, for some integer $k \geq 0$, or
- $w = (aab)^k aa$, for some integer $k \geq 0$.

As always, justify correctness.

Question 6: Professor Justin Bieber claims to have proved the following result:

Bieber's Theorem: Let M be an arbitrary NFA with alphabet $\{a, b\}$ that has exactly one accept state q_f, and let A be the language accepted by M. Let B be the concatenation of A and $\{b\}^*$, i.e.,

$$B = \{vw : v \in A, w \in \{b\}^*\}.$$

Let M' be the NFA obtained by making a copy of M and adding a b-transition from q_f to q_f. Then this new NFA M' accepts the language B.

Is Bieber's Theorem correct? As always, justify your answer.

Question 7: Let A be an arbitrary language over the alphabet $\{a, b\}$. We define the language

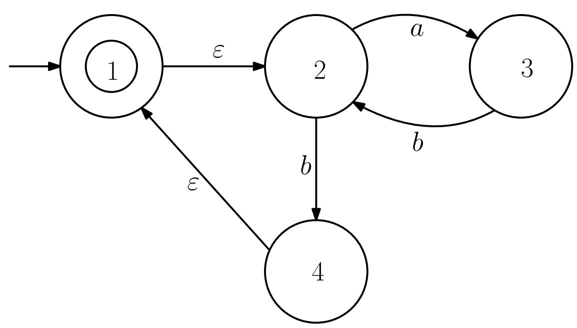
$$A' = \{v \in \{a, b\}^* : \text{there exists a string } w \in A \text{ such that } v \text{ and } w \text{ have the same length and differ in at most one position}\}.$$

For example, if $abba \in A$, then this string gives rise to the five strings $abba$, $bbba$, $aaba$, $abaa$, and $abbb$ in A'.

Prove that if A is regular, then A' is also regular.

Hint: Take a DFA that accepts A. Make multiple copies of its state diagram, and connect the copies with the original state diagram. It is possible to do this without using ε-transitions.

Question 8: Use the construction given in class to convert the following NFA (with alphabet $\{a, b\}$) to an equivalent DFA.



Show the full state diagram of the DFA; it has $2^4 = 16$ states. Afterwards, simplify the diagram by removing states that cannot be reached from the start state (in case this is possible).