COMP 3803 — Assignment 2

Due: Wednesday October 19, 23:59.

Assignment Policy:

- Your assignment must be submitted as one single PDF file through Brightspace.

> Use the following format to name your file:
>
> `LastName_StudentId_a2.pdf`

- **Late assignments will not be accepted. I will not reply to emails of the type "my internet connection broke down at 23:57" or "my scanner stopped working at 23:58", or "my dog ate my laptop charger".**
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

Question 1: Write your name and student number.

Question 2: Consider the language A consisting of all strings over the alphabet $\{a, b\}$ that contain both aba and bab as substrings. Give a regular expression that describes the language A. As always, justify your answer.

Question 3: Let A be the language over the alphabet $\{a, b\}$ that is described by the regular expression aa. Give a regular expression that describes the complement $\overline{A}$ of A. As always, justify your answer.

Question 4: In this question, the alphabet is $\{a, b\}$. A *block* in a string is a maximal substring all of whose symbols are the same. For example, the string *aaabbaa* has three blocks: *aaa*, *bb*, and *aa*.

Let A be the language of all strings w such that every block in w has length two or three. The empty string is in A, as is the string *aaabbaa*.

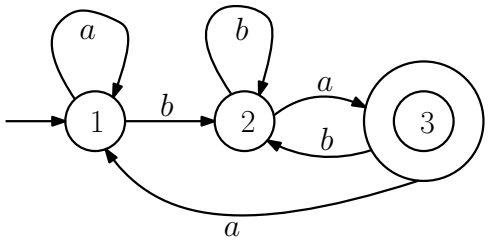
Give a regular expression that describes the language A. As always, justify your answer.

Question 5: Use the construction given in class to convert the regular expression

$$a \cup ba^*$$

to an NFA. Do not simplify your NFA; just apply the construction rules "without thinking".

Question 6: Use the construction given in class to convert the following DFA to a regular expression.



Question 7: Use the Pumping Lemma to prove that the following languages are not regular. In all cases, the alphabet is $\{a, b\}$.

1. $\{a^n u : n \geq 0, u \in \{a, b\}^*, |u| \leq n\}$.
2. $\{a^m b^n : m \geq 0, n \geq 0,\ n \text{ is a multiple of } m\}$.
3. $\{a^m b^n : m \geq 0, n \geq 0,\ m \text{ is a multiple of } n\}$.
   (a) Professor Justin Bieber claims that this can be proven by taking the string $s = a^p b^p$, where p is the pumping length. Show that Professor Bieber is (again!) wrong.
   (b) Now give a correct proof.
4. $\{uv : u \in \{a, b\}^*, v \in \{a, b\}^*, u = v^R\}$.
   Note: If $v = v_1 v_2 \cdots v_n$ is a string, then $v^R = v_n v_{n-1} \cdots v_1$ is the reverse of v.

Question 8: Consider the language

$$A = \{a^{n^2} : n \geq 0\} \cup \{a^{2n+1} : n \geq 0\};$$

note that the alphabet is $\{a\}$.

In this question, you will prove that A is not regular, but the concatenation AA is regular.

1. Prove that A is not a regular language.

   Hint: What is
   $$A \cap \{a^{2n+1} : n \geq 0\}?$$

2. Prove that
   $$AA = \{a^n : n \geq 0\}.$$

3. Prove that AA is a regular language.