COMP 3803 — Assignment 3

Due: Wednesday November 23, 23:59.

Assignment Policy:

- Your assignment must be submitted as one single PDF file through Brightspace.

> Use the following format to name your file:
>
> `LastName_StudentId_a3.pdf`

- **Late assignments will not be accepted. I will not reply to emails of the type "my internet connection broke down at 23:57" or "my scanner stopped working at 23:58", or "my dog ate my laptop charger".**
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

Question 1: Write your name and student number.

Question 2: Consider the context-free grammar $G = (V, \Sigma, R, S)$, where the set of variables is $V = \{S\}$, the set of terminals is $\Sigma = \{a, b\}$, the start variable is S, and the rules are as follows:

$$S \rightarrow \varepsilon \mid aSbS \mid bSaS$$

Determine the language $L(G)$ that is generated by G. Prove that your answer is correct.

(Remember: To prove that two sets X and Y are equal, you have to prove that $X \subseteq Y$ and $Y \subseteq X$.)

Question 3: Give context-free grammars that generate the following languages. For each case, justify your answer.
(3.1) The language of all strings over the alphabet $\{a, b\}$ that have an even number of b's. Note that zero is an even number and ε is in this language.
(3.2) $\{ba^n ba^n b : n \geq 0\}$.

Question 4: A regular expression over the alphabet $\{a, b\}$ can be described by a string whose symbols belong to the set

$$\Sigma = \{\emptyset, \varepsilon, a, b, \cup, ^*, (,)\}.$$

For example, the following string over Σ is a regular expression:

$$((a \cup b)^* ab)^* \cup \emptyset \cup \varepsilon.$$

Give a context-free grammar that generates all regular expressions over the alphabet $\{a, b\}$. The set of terminals of the grammar is Σ.

Question 5: Give (deterministic or nondeterministic) pushdown automata that accept the following languages. For each pushdown automaton, start by explaining the algorithm in plain English, then mention the states that you are going to use, then explain the meaning of these states, and finally give the list of instructions.
(5.1) $\{a^n ba^n : n \geq 0\}$.
(5.2) $\{a^m b^n : 0 \leq n \leq m \leq 2n\}$.

Question 6: Let L_1 and L_2 be two context-free languages over the same alphabet Σ. Assume that you are given a (deterministic or nondeterministic) pushdown automaton M_1 that accepts L_1, and you are also given a (deterministic or nondeterministic) pushdown automaton M_2 that accepts L_2.

Explain how you can combine M_1 and M_2 into one single (deterministic or nondeterministic) pushdown automaton that accepts the concatenation $L_1 L_2$.

Your explanation may be in plain English and may use figures and diagrams.

Question 7: Prove that the following languages are not context-free. In both cases, the alphabet is $\{a, b, c\}$.
(7.1) $\{a^k b^m c^n : 0 \leq k < m < n\}$.
(7.2) $\{a^k b^m c^n : k \geq 0, m \geq 0, n \geq 0, k^2 + m^2 = n^2\}$. *Hint:* $3^2 + 4^2 = 5^2$.