DURATION: 2 HOURS

Department Name & Course Number: **Computer Science COMP 3803A**
Course Instructor: Michiel Smid

**Authorized memoranda:** NONE

Students MUST count the number of pages in this examination question paper before beginning to write, and report any discrepancy to the proctor. This question paper has 13 pages (not including the cover page).

This examination question paper MAY be taken from the examination room.

In addition to this question paper, students require:

- an examination booklet: **no**
- a Scantron sheet: **yes**
Instructions:

1. This is a closed book exam. No aids, notes, or calculating devices are allowed.

2. All questions must be answered on the scantron sheet.

Marking scheme: Each question is worth 1 mark.
1. What is the language of this DFA?

(a) $\{w \in \{0, 1\}^*: w$ starts with 0\}$
(b) $\{w \in \{0, 1\}^*: w$ contains 010\}$
(c) $\{w \in \{0, 1\}^*: w$ starts with 010\}$
(d) $\{w \in \{0, 1\}^*: w$ ends with 010\}$

2. Which of the following four regular expressions describes the language of this NFA?

(a) $0(0 \cup 1)^*0 \cup 1(0 \cup 1)^*1$
(b) $0(0 \cup 1)^*0 \cup 1(0 \cup 1)^*1^*$
(c) $0(0 \cup 1)^*0^* \cup 1(0 \cup 1)^*1$
(d) $0(0 \cup 1)^*0^* \cup 1(0 \cup 1)^*1^*$

3. Let $L = \{a, b\} \cup \{a^n b^n : n \geq 0\}$. What is $L^*$?

(a) $L^* = \{a, b\}^* - \{\epsilon\}$
(b) $L^* = \{a, b\}^*$
(c) $L^* = \{a^n b^n : n \geq 0\}^*$
(d) $L^* = \{a^n b^n : n \geq 1\}^*$
4. Is the following true or false? There exists a regular language that cannot be described by a regular expression.

   (a) True
   (b) False

5. Let \( L, L_1, \) and \( L_2 \) be three languages such that \( L = L_1 \cup L_2 \). Assume that both \( L \) and \( L_1 \) are regular languages. Which of the following is true?

   (a) \( L_2 \) must be regular.
   (b) \( L_2 \) cannot be regular.
   (c) \( L_2 \) may be regular.
   (d) None of the above.

6. Consider the following NFA.

Assume we convert this NFA to an equivalent DFA (without removing unnecessary states). Consider the following statements:

\[ P : \text{the start state of the DFA is } \{1, 2\}. \]
\[ Q : \text{the DFA has 16 accept states.} \]
\[ R : \text{when the DFA is in state } \{5\} \text{ and reads an } a, \text{ it switches to state } \{3, 4\}. \]

Which of the following is correct?

   (a) \( P \) is true, \( Q \) is true, \( R \) is true.
   (b) \( P \) is false, \( Q \) is false, \( R \) is false.
   (c) \( P \) is false, \( Q \) is false, \( R \) is true.
   (d) \( P \) is true, \( Q \) is true, \( R \) is false.
7. Let $L$ be the language 

$$L = \{w \in \{a,b\}^* : \text{every } b \text{ in } w \text{ is followed by an odd number of } a's\}.$$ 

Which of the following regular expressions describes the language $L$?

(a) $(ba(aa)^*)^*$  
(b) $a^* (ba(aa)^*)^*$  
(c) $(baa^*)^*$  
(d) $a^* (baa^*)^*$

8. Consider the following DFA.

For each $i = 1, 2, 3$, let $L_i$ be the language of this DFA if we make $i$ the start state. Consider the following statements:

$P : L_1 = \epsilon \cup aL_3$  
$Q : L_2 = a^*bL_3$  
$R : L_3 = \epsilon \cup bL_1 \cup bL_2 \cup bL_3$

Which of the following is correct?

(a) $P$ is true, $Q$ is true, $R$ is true.  
(b) $P$ is false, $Q$ is false, $R$ is false.  
(c) $P$ is true, $Q$ is false, $R$ is true.  
(d) $P$ is false, $Q$ is true, $R$ is false.
9. Let $A = \{a^n b^k a^m : n > m \geq 0, k \geq 0\}$. Assume we use the Pumping Lemma to prove that $A$ is not a regular language. Which of the following strings can be used to obtain a contradiction? ($p$ denotes the pumping length.)

(a) $a^{p-2} b^2 a^{p-3}$
(b) $a^{p-1} b a^{p-2}$
(c) $a^p b a^{p-1}$
(d) All of the above.

10. Consider the context-free grammar $G = (V, \Sigma, R, S)$, where $V = \{S, A, B\}$ is the set of variables, $\Sigma = \{a, b\}$ is the set of terminals, $S$ is the start variable, and $R$ consists of the following rules:

\[
S \rightarrow ABA \\
A \rightarrow a|bb \\
B \rightarrow \epsilon|bS
\]

Which of the following strings is in the language of this grammar?

(a) $a\ a\ b\ b$
(b) $b\ b\ a\ a\ b\ b$
(c) $b\ b\ b\ a\ b\ b$
(d) None of the above.

11. Consider the context-free grammar $G = (V, \Sigma, R, S)$, where $V = \{S, A\}$ is the set of variables, $\Sigma = \{a, b\}$ is the set of terminals, $S$ is the start variable, and $R$ consists of the following rules:

\[
S \rightarrow a|aS|bA \\
A \rightarrow b|bA
\]

What is the language of this grammar?

(a) $\{a^m : m \geq 1\} \cup \{b^n : n \geq 2\} \cup \{a^m b^n : m \geq 1, n \geq 2\}$
(b) $\{a^m : m \geq 1\} \cup \{b^n : n \geq 1\} \cup \{a^m b^n : m \geq 1, n \geq 1\}$
(c) $\{a^m : m \geq 0\} \cup \{b^n : n \geq 1\} \cup \{a^m b^n : m \geq 0, n \geq 1\}$
(d) $\{a^m : m \geq 0\} \cup \{b^n : n \geq 0\} \cup \{a^m b^n : m \geq 0, n \geq 0\}$
12. Consider the language \( L = \{ a^n b^{2n} : n \geq 1 \} \). Which rules can be used to derive all strings in this language?

(a) \( S \rightarrow \varepsilon | aab | aaSb \)
(b) \( S \rightarrow aab | aaSb \)
(c) \( S \rightarrow abb | aabbbb | aSSbb \)
(d) \( S \rightarrow abb | aSbb \)

13. Consider the pushdown automaton with tape alphabet \( \Sigma = \{ a, b \} \), stack alphabet \( \Gamma = \{ $, S \} \), set of states \( Q = \{ q_1, q_2 \} \), start state \( q_1 \), and instructions

\[
\begin{align*}
q_1a$ & \rightarrow q_1RS (push S) \quad q_2a$ \rightarrow q_2N$ (loop forever) \\
q_1aS & \rightarrow q_1RSS (push S) \quad q_2aS \rightarrow q_2Re (pop) \\
q_1b$ & \rightarrow q_1N$ (loop forever) \quad q_2b$ \rightarrow q_2N$ (loop forever) \\
q_1bS & \rightarrow q_2RS (switch to q_2) \quad q_2bS \rightarrow q_2NS (loop forever) \\
q_1\square$ & \rightarrow q_1N$ (loop forever) \quad q_2\square$ \rightarrow q_2N (accept) \\
q_1\square S & \rightarrow q_1NS (loop forever) \quad q_2\square S \rightarrow q_2NS (loop forever)
\end{align*}
\]

Recall that, at the start of the computation, the stack contains the symbol $ (and nothing else). What is the language of this pushdown automaton?

(a) \( \{ a^n b^2 a^n : n \geq 1 \} \)
(b) \( \{ a^n b a^n : n \geq 0 \} \)
(c) \( \{ a^n b a^n : n \geq 1 \} \)
(d) None of the above.

14. Consider the language \( L = \{ w \in \{ a, b \}^* : \text{the length of } w \text{ is odd, the middle symbol is } a \} \).

Which of the following is correct?

(a) There exists an NFA whose language is equal to \( L \).
(b) There exists a deterministic pushdown automaton whose language is equal to \( L \).
(c) There exists a non-deterministic pushdown automaton whose language is equal to \( L \).
(d) There does not exist a non-deterministic pushdown automaton whose language is equal to \( L \).
15. A 2-	extit{pop} pushdown automaton has all features of the pushdown automata that we have seen in class; additionally, it has instructions of the following type:

- If the automaton is in state $r$, reads a symbol $a$ on the tape, and reads a symbol $S$ on the top of the stack (where $S$ is not the special symbol $\$\, that is always at the bottom of the stack), then the automaton switches to state $r'$, moves one position to the right or does not move on the tape, and \textit{pops the two topmost symbols from the stack}.

Which of the following is true?

(a) A language $L$ can be accepted by a 2-pop pushdown automaton if and only if it can be accepted by a pushdown automaton.

(b) There exists a language $L$ that can be accepted by a 2-pop pushdown automaton but not by a pushdown automaton.

(c) A language $L$ can be accepted by a 2-pop pushdown automaton if and only if it can be accepted by a Turing machine.

(d) There exists a language $L$ that can be accepted by an NFA but not by a 2-pop pushdown automaton.

16. Let $A$ and $B$ be context-free languages. Which of the following is correct?

(a) The language $A \cap B$ must be regular.

(b) The language $A \cap B$ must be context-free.

(c) The language $A \cap B$ cannot be regular.

(d) The language $A \cap B$ may not be context-free.
17. A 2-stack pushdown automaton has one tape and two stacks. Its instructions are of the following type:

- If the automaton is in state $r$, reads a symbol $a$ on the tape, reads a symbol $S_1$ on the top of the first stack, and reads a symbol $S_2$ on the top of the second stack,

- then the automaton switches to state $r'$, moves one position to the right or does not move on the tape, pushes symbols onto or pops the top symbol from the first stack, and pushes symbols onto or pops the top symbol from the second stack.

At the start of the computation, the input string is on the tape with the head at the leftmost symbol, and both stacks contain only the special symbol $. The computation terminates as soon as one of the stacks is empty. The 2-stack pushdown automaton accepts the input string if, after having the read the entire string, both stacks are empty.

Which of the following is true?

(a) There exists a 2-stack pushdown automaton that accepts the language $\{a^n b^n c^n : n \geq 0\}$.

(b) There does not exist a 2-stack pushdown automaton that accepts the language $\{a^n b^n c^n : n \geq 0\}$.

(c) A language $L$ can be accepted by a 2-stack pushdown automaton if and only if it can be accepted by a pushdown automaton.

(d) A language $L$ can be accepted by a 2-stack pushdown automaton if and only if it can be accepted by an NFA.

18. Let $A = \{www : w \in \{a, b\}^*\}$. Assume we use the Pumping Lemma to prove that $A$ is not a context-free language. Which of the following strings can be used to obtain a contradiction? (p denotes the pumping length.)

(a) $s = a^p b^p a^p b^p$.

(b) $s = a^p a^p a^p$.

(c) $s = a^p b^p a^p b^p a^p b^p$.

(d) All of the above.
19. Consider the Turing machine with input alphabet $\Sigma = \{a, b\}$, tape alphabet $\Gamma = \{a, b, \square\}$, set of states $Q = \{q_0, q_1, q_2, q_{accept}, q_{reject}\}$, start state $q_0$, and instructions

$$
\begin{align*}
q_0a & \rightarrow q_1aR \\
q_0b & \rightarrow q_{reject} \\
q_0\square & \rightarrow q_{reject} \\
q_1a & \rightarrow q_1aR \\
q_1b & \rightarrow q_2bR \\
q_1\square & \rightarrow q_{reject} \\
q_2a & \rightarrow q_{reject} \\
q_2b & \rightarrow q_{reject} \\
q_2\square & \rightarrow q_{accept}
\end{align*}
$$

Recall that, at the start of the computation, the tape head is at the leftmost symbol of the input string. Which of the following regular expressions describes the language of this Turing machine?

(a) $a^*b$
(b) $aa^*b$
(c) $a^*b^*$
(d) $aa^*bb^*$

20. Consider the Turing machine with input alphabet $\Sigma = \{a, b\}$, tape alphabet $\Gamma = \{a, b, \square\}$, set of states $Q = \{q_0, q_1, q_2, q_{accept}, q_{reject}\}$, start state $q_0$, and instructions

$$
\begin{align*}
q_0a & \rightarrow q_0aR \\
q_0b & \rightarrow q_0bL \\
q_0\square & \rightarrow q_1\squareL \\
q_1a & \rightarrow q_{reject} \\
q_1b & \rightarrow q_2bL \\
q_1\square & \rightarrow q_{reject} \\
q_2a & \rightarrow q_{reject} \\
q_2b & \rightarrow q_{accept} \\
q_2\square & \rightarrow q_{accept}
\end{align*}
$$

Recall that, at the start of the computation, the tape head is at the leftmost symbol of the input string. What is the language of this Turing machine?

(a) $\{w \in \{a, b\}^* : w \text{ ends with } ab\}$
(b) $\{w \in \{a, b\}^* : w \text{ ends with } bb\} \cup \{\epsilon, b\}$
(c) $\{w \in \{a, b\}^* : w \text{ ends with } bb\}$
(d) $\{w \in \{a, b\}^* : w \text{ ends with } bb\} \cup \{b\}$

21. We proved in class that the Halting Problem is undecidable. Which proof technique did we use?

(a) We used the Pumping Lemma for undecidable languages.
(b) We used induction on the number of states of the NFA that decides the Halting Problem.
(c) We kept on drinking beer until everyone in class agreed that the Halting Problem is undecidable.
(d) We used the same technique that is used to prove that the set of real numbers is not countable.
22. Let $M$ be a Turing machine with input alphabet $\{0, 1\}$ and let $w$ be a binary string. We construct a new Turing machine $T_{Mw}$ which takes as input an arbitrary binary string $x$:

Turing machine $T_{Mw}(x)$:
run $M$ on input $w$;
if $M$ terminates in its accept state
then if $x = 0^n1^n$ for some $n \geq 0$
then terminate in the reject state
else terminate in the accept state
endif
else terminate in the reject state
endif

Recall that $L(T_{Mw})$ denotes the set of all binary strings $x$ that are accepted by the Turing machine $T_{Mw}$. Consider the following statements:

$P$ : If $M$ accepts $w$, then $L(T_{Mw})$ is a regular language.
$Q$ : If $M$ does not accept $w$, then $L(T_{Mw})$ is a regular language.

Which of the following is correct?

(a) $P$ is true, $Q$ is true.
(b) $P$ is true, $Q$ is false.
(c) $P$ is false, $Q$ is true.
(d) $P$ is false, $Q$ is false.

23. Consider the language

$$A = \{ \langle M \rangle : M \text{ is a Turing machine and for every string } w, \text{ when running } M \text{ on input } w, M \text{ is in the start state at least five times during the first 77 computation steps} \}.$$ 

Which of the following is correct?

(a) The language $A$ is decidable.
(b) The language $A$ is undecidable.
24. Let $A$ be a language over the alphabet $\{0, 1\}$ and assume that $A$ is not enumerable. Is the following true or false?

• Since $A$ is not enumerable, $A$ does not have an enumerator, i.e., there does not exist an algorithm that enumerates all strings in $A$. Therefore, $A$ is not countable.

(a) True
(b) False

25. Is the following true or false? Let $A$ be an enumerable language. Then there exists a Turing machine $M$ such that for any input string $w$,

\[ w \in A \text{ if and only if } M \text{ terminates on input } w. \]

(a) True
(b) False