COMP 3803 — Assignment 1

Due: Thursday October 7, 23:59.

Assignment Policy:

- Your assignment must be submitted as one single PDF file through Brightspace.

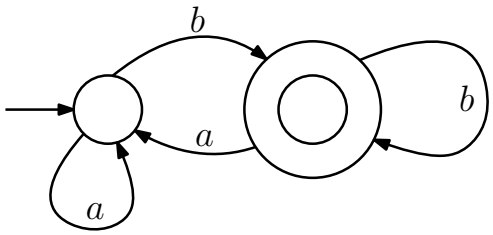
> Use the following format to name your file:
>
> `LastName_StudentId_a1.pdf`

- **Late assignments will not be accepted. I will not reply to emails of the type "my internet connection broke down at 23:57" or "my scanner stopped working at 23:58", or "my dog ate my laptop charger".**
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

> When specifying a finite automaton, it is sufficient to draw the state diagram (because this diagram tells us what are the alphabet, the set of states, the start state, the set of accept states, and the transition function).

Question 1: Write your name and student number.

Question 2: What is the language of the following DFA? The alphabet is $\{a, b\}$. Justify your answer.



Question 3: For each of the following two languages, construct a DFA that accepts the language. In both cases, the alphabet is $\{a, b\}$. For each DFA, justify correctness.
(3.1) $\{abba\}$.
(3.2) $\{w \in \{a, b\}^* : \ w$ does not contain aa and w does not contain $ab\}$.

Question 4: Construct an NFA whose language is the set of all strings $w \in \{a, b\}^*$ such that

- w contains both aa and bb, or
- w does not contain aa and w does not contain bb.

Question 5: Let A be an arbitrary regular language over the alphabet Σ. We define the language

$$A_3 = \{w \in A : \text{ the length of } w \text{ is a multiple of three}\}.$$

(Note that zero is a multiple of three.)

Prove that A_3 is a regular language. You may use any result that was proven in class.

Question 6: Let A be an arbitrary regular language over the alphabet Σ. We define the language

$$A' = \{u \in \Sigma^* : \text{ there exists a string } v \in \Sigma^* \text{ such that } uv \in A\}.$$

(In words, A' consists of all prefixes of strings in A.)

Prove that A' is a regular language.

Hint: Let M be a DFA that accepts A. How would you change the state diagram of M such that the resulting state diagram is a DFA that accepts A'?

Question 7: Let A be an arbitrary regular language over the alphabet Σ. For two strings x and y in Σ^*, we say that the pair (x, y) is *awesome*, if there exists a string z in Σ^* such that (i) $xz \in A$ and $yz \notin A$ or (ii) $xz \notin A$ and $yz \in A$.

Let M be a DFA that accepts the language A, let (x, y) be an awesome pair, let q_x be the state that M is in after having read x, and let q_y be the state that M is in after having read y.

Prove that $q_x \neq q_y$.

Question 8: Use the previous question to prove that the language

$$A = \{a^n b^n : n \geq 0\}$$

is not regular.

Hint: Use a proof by contradiction. If n and m are distinct nonnegative integers, is the pair (a^n, a^m) awesome? You are not allowed to use the Pumping Lemma.

Question 9: Use the construction given in class to convert the following NFA to an equivalent DFA.