# COMP 3803 — Fall 2024 — Assignment 1

**Due:** Friday September 27, 23:59.

**Assignment Policy:**

- Your assignment must be submitted as one single PDF file through Brightspace.

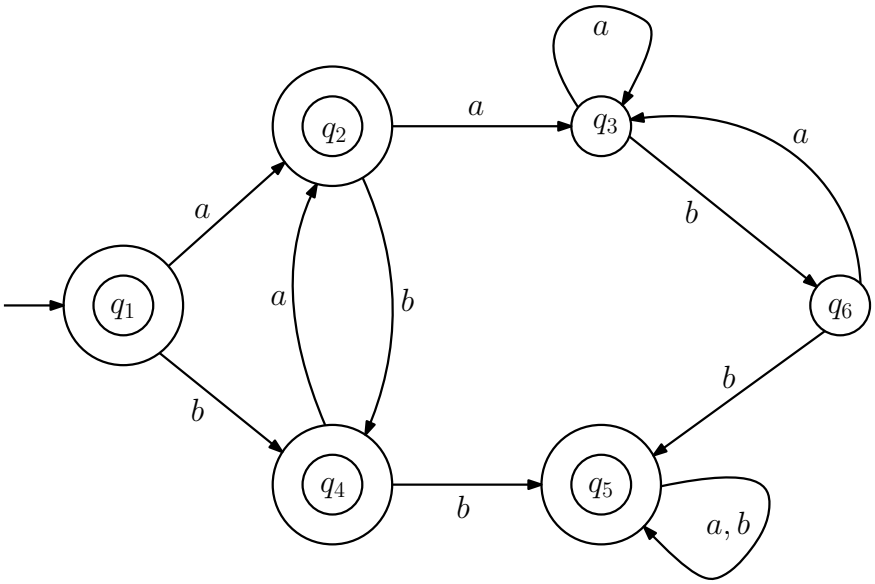
> Use the following format to name your file:
>
> `LastName_StudentId_a1.pdf`

- **Late assignments will not be accepted. I will not reply to emails of the type "my internet connection broke down at 23:57" or "my scanner stopped working at 23:58", or "my dog ate my laptop charger".**
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

> When specifying a finite automaton, it is sufficient to draw the state diagram (because this diagram tells us what are the alphabet, the set of states, the start state, the set of accept states, and the transition function). However, you must explain the meaning of the different states that you use.

**Question 1:** Write your name and student number.

**Question 2:** What is the language of the following DFA? The alphabet is $\{a, b\}$. As always, justify your answer.
*Hint:* After having stared long enough at this DFA, you will know the answer. Once you know the answer, it is not too difficult to prove that your answer is correct.




**Question 3:** Let $A$ be the language consisting of all strings over the alphabet $\{a, b, c\}$ that are alphabetically sorted, i.e., all $a$'s are to the left of all $b$'s, and all $b$'s are to the left of all $c$'s. Note that the empty string $\varepsilon$ is in $A$. Also, strings in $A$ may have no $a$'s or no $b$'s or no $c$'s.

- Construct a DFA with five states that accepts the language $A$. As always, justify your answer.
- Construct an NFA with three states that accepts the language $A$. As always, justify your answer.

**Question 4:** Let $A$ be the language consisting of all strings over the alphabet $\{a, b\}$ that contain $aa$ and do not contain $bb$. Prove that $A$ is a regular language, without explicitly constructing a DFA/NFA that accepts $A$. Instead, use properties that we have seen in class.

**Question 5:** Let $A$ be a regular language over the alphabet $\{a, b\}$. Define the following language

$$B = \{w : \exists\, x \in \{a, b\} \text{ such that } wx \in A\}.$$

In words, $B$ is obtained by deleting the last symbol in every non-empty string in $A$.

Prove that $B$ is a regular language.

*Hint:* Let $M$ be an arbitrary DFA that accepts $A$. Show how to modify $M$ to obtain a DFA that accepts the language $B$.

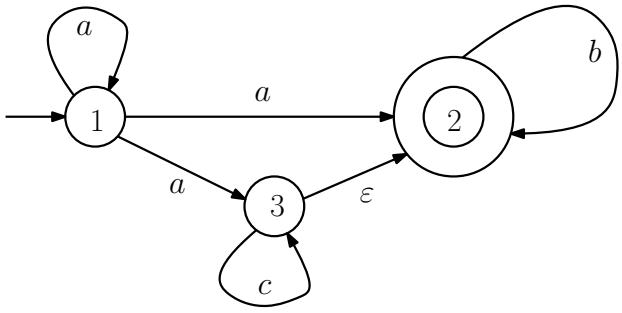
**Question 6:** Let $A$ be an arbitrary regular language and let $M$ be a DFA that accepts $A$. Let $M'$ be the DFA obtained from $M$ by "flipping the states": every accept state in $M$ is a non-accept state in $M'$, and every non-accept state in $M$ is an accept state in $M'$. We have seen in class that $M'$ accepts the complement $\overline{A}$ of $A$.

Professor Taylor Swift claims that this "flipping states" construction also works if $M$ is an NFA that accepts the language $A$.

Is Professor Swift's claim correct? As always, justify your answer.

**Question 7:** Let $A$ be the language consisting of all strings over the alphabet $\{a, b\}$ that have an even length. Construct an NFA with three states that accepts the language $A$ and in which the start state is not an accept state. As always, justify your answer.

**Question 8:** Use the construction given in class to convert the following NFA (with alphabet $\{a, b, c\}$) to an equivalent DFA.




Show the full state diagram of the DFA; it has $2^3 = 8$ states. Afterwards, simplify the diagram by removing states that cannot be reached from the start state (in case this is possible).