COMP 3803 — Assignment 3

Due: Thursday November 25, 23:59.

Assignment Policy:

- Your assignment must be submitted as one single PDF file through Brightspace.

> Use the following format to name your file:
>
> `LastName_StudentId_a3.pdf`

- **Late assignments will not be accepted. I will not reply to emails of the type "my internet connection broke down at 23:57" or "my scanner stopped working at 23:58", or "my dog ate my laptop charger".**
- You are encouraged to collaborate on assignments, but at the level of discussion only. When writing your solutions, you must do so in your own words.
- Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.
- When writing your solutions, you must follow the guidelines below.
  - You must justify your answers.
  - The answers should be concise, clear and neat.
  - When presenting proofs, every step should be justified.

Question 1: Write your name and student number.

Question 2: Consider the context-free grammar $G = (V, \Sigma, R, S)$, where the set of variables is $V = \{S, A, B\}$, the set of terminals is $\Sigma = \{a, b\}$, the start variable is S, and the rules are as follows:

$$\begin{aligned} S &\rightarrow abB \\ A &\rightarrow \epsilon \mid aaBb \\ B &\rightarrow bbAa \end{aligned}$$

Prove that the language $L(G)$ that is generated by G is equal to

$$L(G) = \{ab(bbaa)^n bba(ba)^n \mid n \geq 0\}.$$

(Remember: To prove that two sets X and Y are equal, you have to prove that $X \subseteq Y$ and $Y \subseteq X$.)

Question 3: Give context-free grammars that generate the following languages. For each case, justify your answer.
(3.1) $\{a^{n+3}b^n \mid n \geq 0\}$. The set of terminals is equal to $\{a, b\}$.
(3.2) $\{a^n b^m \mid n \geq 0, m \geq 0, 2n \leq m \leq 3n\}$. The set of terminals is equal to $\{a, b\}$.
(3.3) $\{a^m b^n c^n \mid m \geq 0, n \geq 0\}$. The set of terminals is equal to $\{a, b, c\}$.

Question 4: Give (deterministic or nondeterministic) pushdown automata that accept the following languages. For each pushdown automaton, start by explaining the algorithm in plain English, then mention the states that you are going to use, then explain the meaning of these states, and finally give the list of instructions.
(4.1) $\{0^{2n}1^n \mid n \geq 0\}$.
(4.2) $\{ww^R \mid w \in \{0,1\}^*\}$.
(If $w = w_1 \dots w_n$, then $w^R = w_n \dots w_1$.)

Question 5: Prove that the following languages are not context-free:
(5.1) $\{a^{n!} \mid n \geq 0\}$.
(5.2) $\{a^{n^2} b^n \mid n \geq 0\}$.

Question 6: We have seen that the regular languages are closed under the union, intersection, complement, concatenation, and star operations. In this question, we consider these operations for context-free languages.
(6.1) Let L and L' be context-free languages over the same alphabet Σ. Prove that the union $L \cup L'$ is also context-free.
(6.2) Let L and L' be context-free languages over the same alphabet Σ. Prove that the concatenation LL' is also context-free.
(6.3) Let L be a context-free language over the alphabet Σ. Prove that the star L^* of L is also context-free.
(6.4) In Question 1, you have shown that

$$L = \{a^m b^n c^n \mid m \geq 0, n \geq 0\}$$

is a context-free language. By a symmetric argument, the language

$$L' = \{a^m b^m c^n \mid m \geq 0, n \geq 0\}$$

is context-free.

Prove that the intersection of two context-free languages is not necessarily context-free. (You may use any result that was proven in class.)
(6.5) Prove that the complement of a context-free language is not necessarily context-free.