Carleton University

Final
Examination
Fall 2023

DURATION: 2 HOURS | No. of students: 64

Department Name & Course Number: **Computer Science COMP 3803A**
Course Instructor: Michiel Smid

Authorized memoranda:
NONE

Students MUST count the number of pages in this examination question paper before beginning to write, and report any discrepancy to the proctor. This question paper has 14 pages (not including the cover page).

This examination question paper MAY be taken from the examination room.

In addition to this question paper, students require:

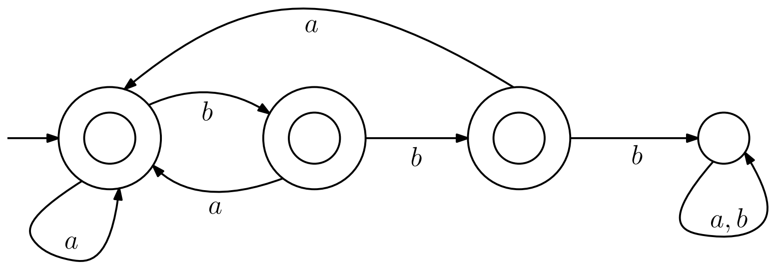
an examination booklet:	**no**
a Scantron sheet:	**yes**

Instructions:

1. This is a closed book exam. No aids, notes, or calculating devices are allowed.
2. All questions must be answered on the scantron sheet.

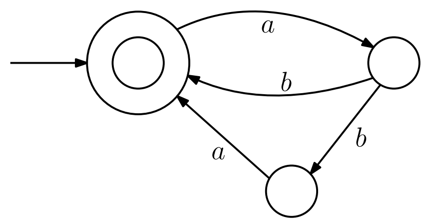
Marking scheme: Each question is worth 1 mark.

1. What is the language of this DFA?



   (a) $\{w \in \{a,b\}^* : \ w \text{ has an even number of } b\text{'s}\}$
   (b) $\{w \in \{a,b\}^* : \ w \text{ has an odd number of } b\text{'s}\}$
   (c) $\{w \in \{a,b\}^* : \ w \text{ does not contain the substring } bbb\}$
   (d) $\{w \in \{a,b\}^* : \ w \text{ contains the substring } bb\}$

2. Which of the following regular expressions describes the language of this NFA?



   (a) $(ab \cup aba)^*$
   (b) $(ba \cup aba)^*$
   (c) $(ab)^*(aba)^*$
   (d) None of the above.

3. Which of the following is true?

   (a) Let L be the language of the NFA M. If we turn every accept state of M into a non-accept state, and turn every non-accept state of M into an accept state, then we obtain an NFA whose language is the complement $\bar{L}$ of L.

   (b) NFA's with ε-transitions can accept languages that cannot be accepted by any NFA without ε-transitions.

   (c) For every regular language L, there exists an NFA whose language is L and that has exactly one accept state.

   (d) There exists a language L consisting of a finite number of strings, such that L is not regular.

4. Let $L = \{a^m b^n : m \geq n \geq 0\}$. Which regular expression describes the language L^*?

   (a) $(a \cup ab)^*$

   (b) $(a \cup aa^*b)^*$

   (c) $(aa^* \cup aa^*b)^*$

   (d) None of the above.

5. Let M be an NFA with alphabet $\{0, 1\}$ that accepts every binary string. Which of the following is true?

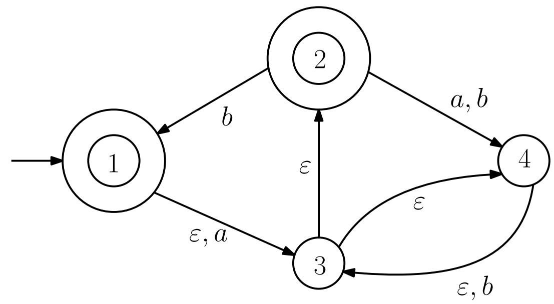
   (a) Every state of M must be an accept state.

   (b) M does not have any accept state.

   (c) The start state of M must be an accept state.

   (d) None of the above.

6. Consider the following NFA.



Assume we convert this NFA to an equivalent DFA (without removing unnecessary states). Consider the following statements:

P : the start state of the DFA is $\{1, 2, 3\}$.
Q : the DFA has 12 accept states.
R : when the DFA is in state $\{4\}$ and reads a b, it switches to state $\{2, 3, 4\}$.

Which of the following is correct?

(a) P is true, Q is true, R is true.
(b) P is false, Q is true, R is true.
(c) P is false, Q is false, R is true.
(d) P is false, Q is false, R is false.

7. Which of the following strings is in the language that is described by the regular expression $((a \cup b)b^*ab^*)^*$.

(a) $aabbabb$
(b) $abbbb$
(c) aaa
(d) $bbaa$

8. Let L be the language

$$L = \{w \in \{a,b\}^* : \text{ every } a \text{ in } w \text{ is followed by at most one } b\}.$$

Which of the following regular expressions describes the language L?

(a) $b^* \left(a^*(\varepsilon \cup b)\right)^*$

(b) $b^* \left(a^* \cup a^*b\right)^*$

(c) $b^* \left(aa^*(\varepsilon \cup b)\right)^*$

(d) None of the above.

9. In this question, the alphabet is $\{a,b\}$. Consider the following statements:

P : Let R_1 be a regular expression that describes the regular language L_1.
Let R_2 be a regular expression that describes the regular language L_2.
Assume that $\varepsilon \in L_1$ and $\varepsilon \in L_2$.
Then $(a \cup b)^*$ and $(R_1(a \cup b)^* R_2)^*$ describe different languages.

Q : The regular expressions $(a \cup b)^*$ and $a^*(ba^*)^*$ describe the same language.

Which of the following is correct?

(a) P is true, Q is true.

(b) P is true, Q is false.

(c) P is false, Q is true.

(d) P is false, Q is false.

10. Let Σ be a non-empty alphabet. Recall the rules that we used to define regular expressions (RE) over this alphabet:

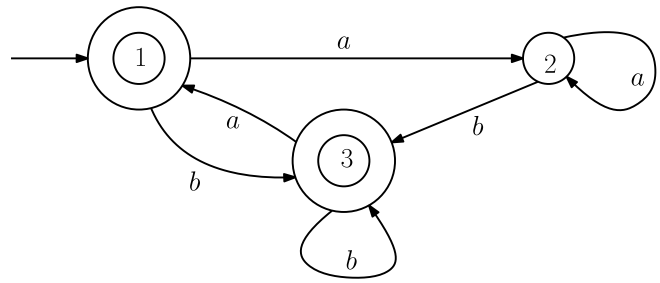
Rule 1: ε is a RE.
Rule 2: $\emptyset$ is a RE.
Rule 3: For each $a \in \Sigma$, a is a RE.
Rule 4: If R_1 and R_2 are RE's, then $R_1 \cup R_2$ is a RE.
Rule 5: If R_1 and R_2 are RE's, then $R_1 R_2$ is a RE.
Rule 6: If R is a RE, then R^* is a RE.

Is the following true or false: We do not need Rule 1, because it follows from Rule 2 and Rule 6.

(a) True

(b) False

11. Consider the following DFA.



For each $i = 1, 2, 3$, let L_i be the language of this DFA if we make i the start state. Consider the following statements:

$$
\begin{aligned}
P &: L_1 = aL_2 \cup bL_3 \\
Q &: L_2 = a^*bL_3 \\
R &: L_3 = b^* \cup b^*aL_1
\end{aligned}
$$

Which of the following is correct?

(a) P is true, Q is true, R is true.

(b) P is false, Q is true, R is true.

(c) P is true, Q is false, R is true.

(d) P is false, Q is true, R is false.

12. Consider the context-free grammar $G = (V, \Sigma, R, S)$, where $V = \{S, X, A, B\}$ is the set of variables, $\Sigma = \{a, b\}$ is the set of terminals, S is the start variable, and R consists of the following rules:

$$\begin{array}{lcl} S & \rightarrow & aXb \\ X & \rightarrow & A|B \\ A & \rightarrow & \varepsilon|aA \\ B & \rightarrow & \varepsilon|bB \end{array}$$

Which of the following strings is in the language of this grammar?

(a) $aaaaab$

(b) $baaaab$

(c) $abaaaab$

(d) $aaabab$

13. Consider the context-free grammar in the previous question. What is the language of this grammar?

(a) $\{a^n b : n \geq 0\} \cup \{ab^n : n \geq 0\}$

(b) $\{b^n a : n \geq 1\} \cup \{ba^n : n \geq 1\}$

(c) $\{w \in \{a, b\} : \ w \text{ does not contain the substring } ba\}$

(d) $\{a^n b : n \geq 1\} \cup \{ab^n : n \geq 1\}$

14. Consider the pushdown automaton with tape alphabet $\Sigma = \{a, b\}$, stack alphabet $\Gamma = \{\$, S\}$, set of states $Q = \{q_0, q_1, q_2\}$, start state q_0, and instructions

$$\begin{array}{lll} q_0 a\$ \rightarrow q_1 R\$S & q_1 aS \rightarrow q_2 R\epsilon & q_2 a\$ \rightarrow q_2 R\$ \\ q_0 b\$ \rightarrow q_0 R\$ & q_1 bS \rightarrow q_0 R\epsilon & q_2 b\$ \rightarrow q_2 R\$ \\ q_0 \square\$ \rightarrow q_0 N\$ & q_1 \square S \rightarrow q_1 NS & q_2 \square\$ \rightarrow q_2 N\epsilon \end{array}$$

Recall that, at the start of the computation, the stack contains the symbol \$ (and nothing else). Which of the following strings is accepted by this pushdown automaton?

(a) ϵ

(b) $baba$

(c) $babaab$

(d) $abba$

15. Consider the pushdown automaton in the previous question. What is the language of this pushdown automaton?

(a) $\{w \in \{a, b\}^* : w \text{ contains the substring } bb\}$

(b) $\{w \in \{a, b\}^* : w \text{ does not contain the substring } aa\}$

(c) $\{w \in \{a, b\}^* : w \text{ contains the substring } ba\}$

(d) $\{w \in \{a, b\}^* : w \text{ contains the substring } aa\}$

16. Recall that, for any string $w = w_1w_2 \dots w_{n-1}w_n$, we denote by w^R the string obtained by reversing w. Thus, $w^R = w_nw_{n-1} \dots w_2w_1$.
Consider the following statements:

P : There exists a deterministic pushdown automaton whose language is $\{wcw^R : w \in \{a,b\}^*\}$. Here, the alphabet is $\{a,b,c\}$.

Q : There exists a nondeterministic pushdown automaton whose language is $\{ww^R : w \in \{a,b\}^*\}$. Here, the alphabet is $\{a,b\}$.

Which of the following is correct?

(a) P is true, Q is false.

(b) P is true, Q is true.

(c) P is false, Q is true.

(d) P is false, Q is false.

17. Let $L = \{www : w \in \{a,b\}^*\}$. Assume we use the Pumping Lemma to prove that L is not a context-free language. Which of the following strings can be used to obtain a contradiction? (p denotes the pumping length.)

(a) $s = a^pb^pa^pb^p$.

(b) $s = a^pb^pa^pb^pa^pb^p$.

(c) $s = a^pa^pa^p$.

(d) All of the above.

18. Consider the Turing machine with input alphabet $\Sigma = \{a, b\}$, tape alphabet $\Gamma = \{a, b, \Box\}$, set of states $Q = \{q, q_{accept}, q_{reject}\}$, start state q, and instructions

$$\begin{array}{lcl} qa & \rightarrow & qaR \\ qb & \rightarrow & qbL \\ q\Box & \rightarrow & q_{accept} \end{array}$$

Recall that, at the start of the computation, the tape head is at the leftmost symbol of the input string. Which of the following regular expressions describes the language of this Turing machine?

(a) $a^* \cup b(a \cup b)^*$

(b) $a^*b \cup b(a \cup b)^*$

(c) $ba^* \cup b(a \cup b)^*$

(d) $(a \cup b)^*$

19. Consider the Turing machine with input alphabet $\Sigma = \{a, b\}$, tape alphabet $\Gamma = \{a, b, \Box\}$, set of states $Q = \{q_0, q_1, q_{accept}, q_{reject}\}$, start state q_0, and instructions

$$\begin{array}{lcl} q_0a & \rightarrow & q_1aR \\ q_0b & \rightarrow & q_0bR \\ q_0\Box & \rightarrow & q_{reject} \\ q_1a & \rightarrow & q_{accept} \\ q_1b & \rightarrow & q_0bR \\ q_1\Box & \rightarrow & q_{reject} \end{array}$$

Recall that, at the start of the computation, the tape head is at the leftmost symbol of the input string. What is the language of this Turing machine?

(a) $\{w \in \{a, b\}^* : w \text{ contains the substring } bb\}$

(b) $\{w \in \{a, b\}^* : w \text{ contains the substring } ab\}$

(c) $\{w \in \{a, b\}^* : w \text{ contains the substring } ba\}$

(d) $\{w \in \{a, b\}^* : w \text{ contains the substring } aa\}$

20. Consider the language L consisting of all pairs $\langle P, x\rangle$, encoded as bitstrings, where P is a Java program and x is an integer (written in binary notation), such that on input x, the program P terminates and returns the integer $x + 1$.
Which of the following is true?

    (a) The language L is decidable.
    (b) The language L is undecidable.

21. Let M be a Turing machine with input alphabet $\{0, 1\}$ and let w be a binary string.
We construct a new Turing machine T_{Mw} which takes as input an arbitrary binary string x:

    Turing machine $T_{Mw}(x)$:
    if $x = 0^n 1^n$ for some $n \geq 0$
    then terminate in the accept state
    else run M on the input string w;
        if M terminates in the accept state
        then terminate in the accept state
        else if M terminates in the reject state
            then terminate in the reject state
            endif
        endif
    endif

    Recall that $L(T_{Mw})$ denotes the set of all binary strings x that are accepted by the Turing machine T_{Mw}. Consider the following statements:

    P : If M accepts w, then $L(T_{Mw})$ is not a regular language.
    Q : If M does not accept w, then $L(T_{Mw})$ is a regular language.

    Which of the following is correct?

    (a) P is true, Q is true.
    (b) P is true, Q is false.
    (c) P is false, Q is true.
    (d) P is false, Q is false.

22. Consider the following statements:

P : For every decidable language L, the complement of L is also decidable.
Q : For every enumerable language L, the complement of L is also enumerable.

Which of the following is correct?

(a) P is true, Q is true.
(b) P is true, Q is false.
(c) P is false, Q is true.
(d) P is false, Q is false.

23. Consider the language

$$L = \{\langle G, w\rangle : G \text{ is a context-free grammar and } w \in L(G)\}.$$

Which of the following is correct?

(a) The language L is decidable.
(b) The language L is undecidable.

24. Let L be a language over the alphabet $\{a\}$. Is the following true or false? If the language L^* is context-free, then the language L is also context-free.

(a) True
(b) False

25. Let L be the language over the alphabet $\{a, b, c\}$ consisting of all strings of the form $x\,c^n$, where $x \in \{a, b\}^*$, $n \geq 0$, and the number of a's in the string x is equal to n.
Which of the following is true?

(a) L is context-free.
(b) L is not context-free.