Midterm COMP 2805

October 24, 2011

Student Name:

Student Number:

• All questions must be answered on this examination paper as well as on the scantron sheet.

Marking scheme: Each question is worth 2 marks, except the last one, which is worth 1 mark.
1. Which of the following strings is accepted by this DFA?

(a) $\epsilon$
(b) 0011
(c) 111000
(d) 1100

2. Consider the DFA in the previous question. What is the language of this DFA?

(a) $\{w \in \{0, 1\}^* : w \text{ has even length}\}$
(b) $\{w \in \{0, 1\}^* : w \text{ has odd length}\}$
(c) $\{w \in \{0, 1\}^* : w \text{ has an odd number of 0's}\}$
(d) $\{w \in \{0, 1\}^* : w \text{ ends with an odd number of 0's}\}$, i.e., either $w$ has an odd number of 0's and no 1's, or there are an odd number of 0's after the rightmost 1.
3. Which of the following strings is accepted by this NFA?

(a) 00000 (i.e., five 0’s)
(b) 000 (i.e., three 0’s)
(c) 0000000 (i.e., seven 0’s)
(d) 00000000000 (i.e., eleven 0’s)

4. Consider the NFA in the previous question. What is the language of this NFA?

(a) \{0\}^*
(b) \{w \in \{0\}^* : \text{the length of } w \text{ is odd}\}
(c) \{w \in \{0\}^* : \text{the length of } w \text{ is even and a multiple of three}\}
(d) \{w \in \{0\}^* : \text{the length of } w \text{ is even or a multiple of three}\}
5. Let $M$ be an NFA that accepts the empty string $\epsilon$. (In other words, $\epsilon \in L(M)$. ) Which of the following is true?

   (a) The start state of $M$ must be an accept state.
   (b) The start state of $M$ cannot be an accept state.
   (c) There is an NFA that accepts the same language as $M$ and that has exactly one accept state.
   (d) In $M$, there must be an $\epsilon$-transition from the start state to every accept state.

6. Let $M$ be an NFA with alphabet $\{0, 1\}$ that accepts every binary string. Which of the following is true?

   (a) Every state of $M$ must be an accept state.
   (b) $M$ does not have any accept state.
   (c) The start state of $M$ must be an accept state.
   (d) None of the above.
7. Consider the following NFA.

Assume we convert this NFA to an equivalent DFA (without removing unnecessary states). Consider the following statements:

- \( P \) : the start state of the DFA is \( \{1, 2, 3\} \).
- \( Q \) : the DFA has 24 accept states.
- \( R \) : when the DFA is in state \( \{5\} \) and reads an \( a \), it switches to the state \( \{1, 2, 3, 4, 5\} \).

Which of the following are correct?

(a) \( P \) is true, \( Q \) is true, \( R \) is false.
(b) \( P \) is false, \( Q \) is false, \( R \) is true.
(c) \( P \) is true, \( Q \) is true, \( R \) is true.
(d) \( P \) is false, \( Q \) is true, \( R \) is false.

8. Which of the following strings is in the language that is described by the regular expression \((ab^*a \cup b^*)^*\).

(a) \( aaa \)
(b) \( bbabaaaa \)
(c) \( aaba \)
(d) \( baaba \)
9. Let $A$ be the language 

$$A = \{w \in \{0, 1\}^* : w \text{ does not contain the substring } 10\}.$$ 

Which of the following regular expressions describes the language $A$?

(a) $0^*1^*$
(b) $(0 \cup 1)^*10(0 \cup 1)^*$
(c) $(0 \cup 1)^*00(0 \cup 1)^* \cup (0 \cup 1)^*01(0 \cup 1)^* \cup (0 \cup 1)^*11(0 \cup 1)^*$
(d) $(0 \cup 1)^*(0 \cup 1)^*$

10. Consider the following DFA.

For each $i = 1, 2, 3$, let $L_i$ be the language of this DFA if we make $i$ the start state. Consider the following statements:

$P : L_1 = aL_2 \cup bL_3$

$Q : L_2 = a^*bL_3$

$R : L_3 = b^*(\epsilon \cup aL_1)$

Which of the following are correct?

(a) $P$ is true, $Q$ is true, $R$ is true.
(b) $P$ is true, $Q$ is false, $R$ is true.
(c) $P$ is false, $Q$ is true, $R$ is true.
(d) $P$ is false, $Q$ is true, $R$ is false.

11. Let $A = \{a^m b^n : m \geq n \geq 0\}$. Assume we use the pumping lemma to prove that $A$ is not a regular language. Which of the following strings can be used to obtain a contradiction? ($p$ denotes the pumping length.)

(a) $s = a^pb^p$.
(b) $s = a^{p-1}b^p$.
(c) $s = a^pb^{p-1}$.
(d) $s = a^mb^m$. 
12. Let $A$ and $B$ be languages such that $A \subseteq B$; thus, $A$ is a subset of $B$. Assume that the language $B$ is regular. Which of the following is true?

(a) $A$ must be regular.
(b) $A$ cannot be regular.
(c) $A$ may be regular.
(d) Since the pumping lemma applies to $B$, and since $A \subseteq B$, the pumping lemma applies to $A$ as well.

13. Did you write your name and student number on this examination paper as well as on the scantron sheet?

(a) Yes.