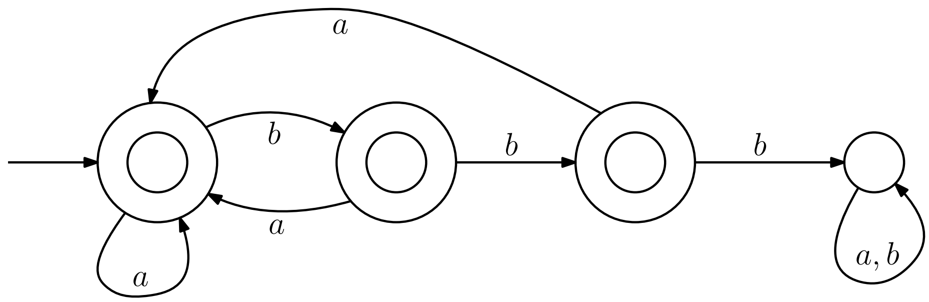
COMP 3803 — Fall 2025 — Problem Set 1

Question 1: What is the language of the following DFA? The alphabet is $\{a, b\}$. As always, justify your answer.



Question 2: Construct a DFA that accepts the language

$$A = \{auaava : u \text{ and } v \text{ are in } \{a, b\}^*\}.$$

The alphabet is $\{a, b\}$. As always, justify your answer.

Question 3: Let A be a regular language over the alphabet $\Sigma = \{a, b\}$, and assume that the empty string ϵ is contained in A. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA that accepts A.

Show how to modify M to obtain a DFA that accepts the language $A \setminus \{\varepsilon\}$. As always, justify your answer.

Question 4: In class, we have seen the following three operations:

- If A and B are languages over the same alphabet, then their *union* is the language

$$A \cup B = \{w : w \in A \text{ or } w \in B\}.$$

- If A and B are languages over the same alphabet, then their *concatenation* is the language

$$AB = \{uv : u \in A \text{ and } v \in B\}.$$

- If A is a language, then the *star* of A is the language

$$A^* = \{u_1 u_2 \dots u_k : k \geq 0 \text{ and } u_i \in A \text{ for all } i = 1, 2, \dots, k\}.$$

Professor Justin Bieber claims that the following two facts imply that the star of a regular language is regular:

Fact 1: The union of two regular languages is regular.

Fact 2: The concatenation of two regular languages is regular.

Professor Bieber's Proof: Let A be an arbitrary regular language. We define the languages A^k, for $k \geq 0$, recursively as follows:

- $A^0 = \{\varepsilon\}$.
- For $k \geq 1$, $A^k = AA^{k-1}$, i.e., A^k is the concatenation of A and A^{k-1}.

Is is obvious that A^0 is regular, because there exists a DFA that only accepts the empty string ε. From this, a straightforward induction proof, that uses **Fact 2** in the induction step, implies that A^k is regular for every $k \geq 0$.

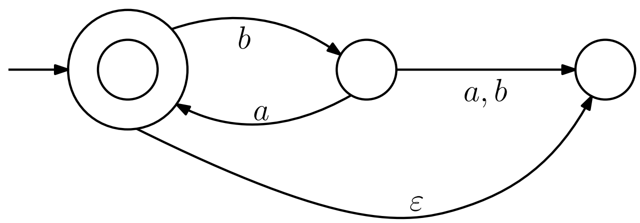
Again, using a straightforward induction proof, that uses **Fact 1** in the induction step, we see that for every integer $k \geq 0$,

$$A^0 \cup A^1 \cup A^2 \cup \cdots \cup A^k \tag{1}$$

is regular. Since (1) holds for all integers $k \geq 0$, we conclude that A^* is regular. QED

Is Professor Bieber's Proof correct? As always, justify your answer.

Question 5: What is the language of the following NFA? The alphabet is $\{a, b\}$. As always, justify your answer.



Question 6: Construct an NFA with three states that accepts the language

$$A = \{ab, abc\}^*.$$

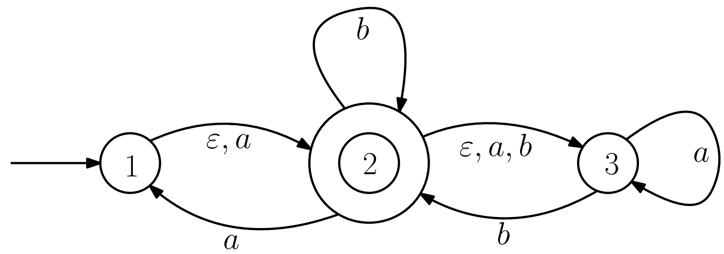
The alphabet is $\{a, b, c\}$. As always, justify your answer.

Question 7: Construct an NFA M with alphabet $\{a\}$ that has the following two properties:

- M accepts the language $\{a\}^*$.
- There is a transition in the state diagram of M, such that the language of the NFA obtained by removing this transition is equal to $\{a\}$.

As always, justify your answer.

Question 8: Use the construction given in class to convert the following NFA (with alphabet $\{a, b\}$) to an equivalent DFA.



Show the full state diagram of the DFA; it has $2^3 = 8$ states. Afterwards, simplify the diagram by removing states that cannot be reached from the start state (in case this is possible).