COMP 3803 — Fall 2025 — Problem Set 4

Question 1: Give (deterministic or nondeterministic) pushdown automata that accept the following languages. For each pushdown automaton, start by explaining the algorithm in plain English, then mention the states that you are going to use, then explain the meaning of these states, and finally give the list of instructions.
(1.1) $\{a^{2n}b^n : n \geq 0\}$.
(1.2) $\{ww^R : w \in \{a, b\}^*\}$.
(If $w = w_1 \dots w_n$, then $w^R = w_n \dots w_1$. Note that $\varepsilon^R = \varepsilon$.)
(1.3) $\{a^n : n \geq 0\} \cup \{a^n b^n : n \geq 0\}$.
(1.4) $\{a^m b^n : m \geq 0, n \geq 0, m \neq n\}$.

Question 2: Prove that the following languages are not context-free:
(2.1) $\{a^{n!} : n \geq 0\}$.
(2.2) $\{a^{n^2} b^n : n \geq 0\}$.
(2.3) $\{a^m b^n a^m b^n : m \geq 0, n \geq 0\}$.
(2.4) $\{a^m b^n c^k : 1 \leq m < n < k < 2m\}$.

Question 3: We have seen that the regular languages are closed under the union, intersection, complement, concatenation, and star operations. In this question, we consider these operations for context-free languages.
(3.1) Let L and L' be context-free languages over the same alphabet Σ. Prove that the union $L \cup L'$ is also context-free.
(3.2) Let L and L' be context-free languages over the same alphabet Σ. Prove that the concatenation LL' is also context-free.
(3.3) Let L be a context-free language over the alphabet Σ. Prove that the star L^* of L is also context-free.
(3.4) Prove that both

$$L = \{a^m b^n c^n : m \geq 0, n \geq 0\}$$

and

$$L' = \{a^m b^m c^n : m \geq 0, n \geq 0\}$$

are context-free languages.
(3.5) Prove that the intersection of two context-free languages is not necessarily context-free.
(3.6) Prove that the complement of a context-free language is not necessarily context-free.