Carleton University
COMP 3803, Fall 2025, Solutions Test 3

Thursday November 20, 2025

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40 marks total

Question 1: (**10 marks**) Consider the pushdown automaton with tape alphabet $\Sigma = \{a, b\}$, stack alphabet $\Gamma = \{\$, S\}$, set of states $Q = \{q_0, q_1, q_2\}$, start state q_0, and instructions

$$\begin{array}{lclclclclcl} q_0 a\$ & \rightarrow & q_1 R\$S & \quad & q_1 aS & \rightarrow & q_2 R\epsilon & \quad & q_2 a\$ & \rightarrow & q_2 R\$ \\ q_0 b\$ & \rightarrow & q_0 R\$ & & q_1 bS & \rightarrow & q_0 R\epsilon & & q_2 b\$ & \rightarrow & q_2 R\$ \\ q_0 \Box\$ & \rightarrow & q_0 N\$ & & q_1 \Box S & \rightarrow & q_1 NS & & q_2 \Box\$ & \rightarrow & q_2 N\epsilon \end{array}$$

Recall that, at the start of the computation, the stack contains the symbol \$ (and nothing else).

Exactly one of the following four strings is accepted by this pushdown automaton.

- ϵ
- *baba*
- *babaab*
- *abba*

Which one is it? Just give the answer. No explanation is needed.

Solution: The correct answer is *babaab*.

Question 2: (**10 marks**) Let L be the language

$$L = \{(ba)^n a^n : n \geq 0\}.$$

Give an **informal** description of a pushdown automaton that accepts the language L. (Describe the algorithm in plain English. Do not give the instructions.)

Solution:

1. If the leftmost symbol is $\Box$: empty the stack, terminate, and accept.
2. If the leftmost symbol is a: loop forever.
3. If the leftmost symbol is b: switch to the state "we just read b", push(S), move right. (Note: the number of S on the stack is equal to the number of b's we have read.)
   (a) If the next symbol is b or $\Box$: loop forever.
   (b) If the next symbol is a: switch to the state "we just read ba", do not change the stack, move right.
      i. If the next symbol is $\Box$: loop forever.
      ii. If the next symbol is b: proceed as in 3.
      iii. If the next symbol is a: switch to the state "we are in the a^n-block", pop the top symbol from the stack, move right.
         A. If the next symbol is b: loop forever.
         B. If the next symbol is a and the stack has S on top: stay in the state "we are in the a^n-block", pop the top symbol from the stack, move right.
         C. If the next symbol is a and the stack has \$ on top: loop forever.
         D. If the next symbol is $\Box$ and the stack has S on top: loop forever.
         E. If the next symbol is $\Box$ and the stack has \$ on top: empty the stack, terminate, and accept.

Question 3: (**10 marks**) Let L be the language

$$L = \{ww : w \in \{a,b\}^*\}.$$

We want to apply the Pumping Lemma for context-free languages to prove that L is not context-free: As always, we assume that L is context-free. Let p be the pumping length, as given by the Pumpng Lemma.

Can we use the string $s = a^p b a^p b$ to obtain a contradiction?

Justify your answer.

Solution: This was done in class.

First note that $s \in L$ and $|s| = 2p + 2 \geq p$. Thus, we can write $s = uvxyz$, where $|vy| \geq 1$, $|vxy| \leq p$, and for all $i \geq 0$, the string $uv^i xy^i z$ is in L.

It may happen that $p \geq 3$, $u = a^{p-1}$, $v = a$, $x = b$, $y = a$, and $z = a^{p-1}b$. If this is the case, then

$$uv^i xy^i z = a^{p-1} a^i b a^i a^{p-1} b = a^{p+i-1} b a^{p+i-1} b,$$

which is in L for any integer $i \geq 0$. Thus, this is the "wrong" string; we do not obtain a contradiction.

Question 4: (**10 marks**) Let L be the language

$$L = \{a^m b^n c^k : m \geq 0, n \geq 0, k = \max(m, n)\}.$$

Prove that L is not a context-free language.

Solution: Assume that L is context-free. Let $p \geq 1$ be the pumping length.

Take the string $s = a^p b^p c^p$. Then $s \in L$ and $|s| = 3p \geq p$. Thus, we can write $s = uvxyz$, where $|vy| \geq 1$, $|vxy| \leq p$, and for all $i \geq 0$, the string $uv^i xy^i z$ is in L.

There are two possible cases.

Case 1: vxy does not contain c.

Consider the string uv^2xy^2z. The number of c's in this string is equal to p. However, there are at least $p + 1$ many a's or at least $p + 1$ many b's. Thus, this string is not in L, which is a contradiction.

Case 2: vxy contains c.

Since $|vxy| \leq p$, the string vxy does not contain a.

Consider the string $uv^0xy^0z = uxz$. The number of a's in this string is equal to p. However, there are at most $p - 1$ many c's. Thus, this string is not in L, which is a contradiction.

EXTRA PAGE FOR ANSWERS