Carleton University
COMP 3803, Fall 2025, Test 3

Thursday November 20, 2025

Student Name:

Student Number:

40 marks total

Question 1: (**10 marks**) Consider the pushdown automaton with tape alphabet $\Sigma = \{a, b\}$, stack alphabet $\Gamma = \{\$, S\}$, set of states $Q = \{q_0, q_1, q_2\}$, start state q_0, and instructions

$$
\begin{array}{lcllcllcl}
q_0a\$ & \to & q_1R\$S & q_1aS & \to & q_2R\epsilon & q_2a\$ & \to & q_2R\$ \\
q_0b\$ & \to & q_0R\$ & q_1bS & \to & q_0R\epsilon & q_2b\$ & \to & q_2R\$ \\
q_0\Box\$ & \to & q_0N\$ & q_1\Box S & \to & q_1NS & q_2\Box\$ & \to & q_2N\epsilon
\end{array}
$$

Recall that, at the start of the computation, the stack contains the symbol \$ (and nothing else).

Exactly one of the following four strings is accepted by this pushdown automaton.

- ϵ
- *baba*
- *babaab*
- *abba*

Which one is it? Just give the answer. No explanation is needed.

Question 2: (**10 marks**) Let L be the language

$$L = \{(ba)^n a^n : n \geq 0\}.$$

Give an **informal** description of a pushdown automaton that accepts the language L. (Describe the algorithm in plain English. Do not give the instructions.)

Question 3: (**10 marks**) Let L be the language

$$L = \{ww : w \in \{a, b\}^*\}.$$

We want to apply the Pumping Lemma for context-free languages to prove that L is not context-free: As always, we assume that L is context-free. Let p be the pumping length, as given by the Pumpng Lemma.

Can we use the string $s = a^p b a^p b$ to obtain a contradiction?

Justify your answer.

Question 4: (**10 marks**) Let L be the language

$$L = \{a^m b^n c^k : m \geq 0, n \geq 0, k = \max(m, n)\}.$$

Prove that L is not a context-free language.

EXTRA PAGE FOR ANSWERS