COMP 3804 — Tutorial January 24, 2025

Problem 1: In the median-of-median algorithm, we divided the length-n input sequence into $n/5$ groups, each of length 5.

1. Assume we use $n/7$ groups, each of length 7. State the recurrence for the running time $T(n)$ of the resulting algorithm. What is the solution to this recurrence?

2. Assume we use $n/3$ groups, each of length 3. State the recurrence for the running time $T(n)$ of the resulting algorithm. What is the solution to this recurrence?

Problem 2: In class, you have seen a randomized algorithm that computes the k-th smallest element in a sequence of n numbers. In the analysis, we considered different *phases* of this algorithm. Recall that a call to the algorithm is in phase i if it is on a sequence with more than $(3/4)^{i+1} \cdot n$ numbers and at most $(3/4)^i \cdot n$ numbers.

1. What is the number of phases?

2. In the analysis of the expected running time, we never used the number of phases. Can you explain why?

Problem 3: You are given an unlimited supply of identical egges. A natural question is: What is the largest integer n, such that the eggs do not break if you drop them from the n-th floor of a building.

Let's phrase this more carefully. You take the eggs to a building that has an unlimited number of floors. These floors are numbered $0, 1, 2, \ldots$; the ground floor has number 0.

- If you drop an egg from the 0-th floor, it does not break.

- For any integer $k \geq 1$: if an egg does not break by dropping it from the k-th floor, then it does not break if you drop it from any lower floor.

- For any integer $k \geq 1$: if an egg breaks by dropping it from the k-th floor, then it also breaks if you drop it from any higher floor.

- You are told that there is an integer n, such that eggs do not break if you drop them from any of the floors $0, 1, \ldots, n$, but eggs do break if you drop them from any of the floors $n + 1, n + 2, \ldots$

Your task is to determine this integer n. The only way to do this is by dropping eggs from some carefully chosen floors.

1. Assume you are allowed to break only one egg. What is the smallest number of eggs you have to drop, so that you are guaranteed to determine the value of n?

2. Assume you are allowed to break at most two eggs. What is the smallest number of eggs you have to drop, so that you are guaranteed to determine the value of n?

3. Let k be a large integer. Assume you are allowed to break at most $k + c$ eggs, where c is a small constant. Prove that you can determine the value of n by dropping at most $\frac{n}{2^k} + O(k)$ eggs.

4. Prove that you can determine the value of n by dropping only $O(\log n)$ eggs. (Note: You cannot take k to be $\log n$ in the previous part. Why not?)