

COMP 3804 — Tutorial January 31, 2025

Problem 1: Some algorithms textbooks have statements of the type

Every comparison-based sorting algorithm takes at least $O(n \log n)$ time.

Does such a statement make sense?

Problem 2: Let $A[1 \dots N]$ be an array storing N numbers. In class, we have seen algorithm $\text{BUILDHEAP}(A)$ that rearranges the numbers in the input array A such that the resulting array is a max-heap; see page 56 of my handwritten notes. This algorithm uses the HEAPIFY -procedure as a subroutine; see page 53 of my handwritten notes. Consider the following variant of this algorithm:

```
Algorithm BUILDHEAP'(A):  
for  $i = 1$  to  $\lfloor N/2 \rfloor$   
do HEAPIFY( $A, i$ )  
endfor
```

Give an example of an array $A[1 \dots N]$, where N is a small integer (such as $N = 7$), which shows that algorithm $\text{BUILDHEAP}'$ may not result in a max-heap.

Problem 3: Let $A[1 \dots n]$ be an array storing n pairwise distinct numbers, and let k be an integer with $0 \leq k < n$. We say that this array is k -sorted, if for each i with $1 \leq i \leq n$, the entry $A[i]$ is at most k positions away from its position in the sorted order.

For example, a sorted array is 0-sorted. As another example, the array

$$A[1 \dots 10] = [1, 4, 5, 2, 3, 7, 8, 6, 10, 9]$$

is 2-sorted, because each entry $A[i]$ is at most 2 positions away from its position in the sorted order. For $i = 3$, $A[3]$ is 2 positions away from its position, 5, in the sorted array. For $i = 9$, $A[9]$ is 1 position away from its position, 10, in the sorted array.

Describe an algorithm SORT that has the following specification:

```
Algorithm SORT( $A, k$ ):  
Input: An array  $A[1 \dots n]$  of  $n$  pairwise distinct numbers and an integer  $k$  with  $2 \leq k < n$ . This array is  $k$ -sorted.  
Output: An array  $B[1 \dots n]$  containing the same numbers as the input array. The array  $B$  is sorted.  
Running time: Must be  $O(n \log k)$ .
```

Explain why your algorithm is correct and why the running time is $O(n \log k)$.

Hint: Use a min-heap of a certain size.