Consider the following recurrence:

\[ T(1) = 1. \]

For \( n \geq 2 \),

\[ T(n) = n + T(\lfloor n/9 \rfloor) + T(\lfloor 8n/9 \rfloor). \]

Which of the following is correct?

- \( T(n) = \Theta(n^{\log_9 8}). \)
- \( T(n) = \Theta(n^2). \)
- \( T(n) = \Theta(n \log n). \)
- \( T(n) = \Theta(n). \)
Question 2 (1 point)

Professor Lionel Messi has an algorithm that solves a problem of input size $n$, by

recursively solving $27$ problems, each of input size $n/3$, and

spending $\Theta(n^3)$ additional time.

What is the running time of Professor Messi's algorithm?

- $\Theta(n^3 \log n)$.
- None of the other answers.
- $\Theta(n^3)$.
- $\Theta(n^{\log_{27} 3})$. 

Submit Quiz  0 of 25 questions saved
Let $A[1 \ldots n]$ be an array storing pairwise distinct numbers, and let $k$ be an integer with $1 \leq k \leq n$.

What is the running time of the fastest possible algorithm that returns the $k$ smallest elements in the array $A$?

- $\Theta(n \log n)$.
- $\Theta(n \log k)$.
- $\Theta(k \log n)$.
- $\Theta(n)$. 

Submit Quiz 0 of 25 questions saved
Consider the following algorithm that takes as input an integer $n \geq 1$:

Algorithm $\text{HelloWorld}(n)$:

If $n = 1$: print "Hello World"

Else: Let $k = \lfloor \sqrt{n} \rfloor$; $\text{HelloWorld}(k)$

End of algorithm

Let $n$ be a large integer. What is the running time of algorithm $\text{HelloWorld}(n)$?

- $\Theta(1)$.
- $\Theta(\log \log n)$.
- $\Theta(\log n)$.
- $\Theta(\sqrt{n})$. 

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Let $A[1, \ldots, n]$ be a max-heap storing $n$ pairwise distinct numbers, and let $x$ be the 5-th largest number stored in $A$.

Which of the following is true?

- $x$ can be stored at any of the positions $2, 3, \ldots, 31$.
- $x$ can be stored at any of the positions $2, 3, \ldots, 63$.
- $x$ can be stored at any of the positions $5, 6, \ldots, 31$.
- $x$ can be stored at any of the positions $5, 6, \ldots, 63$. 
Question 6 (1 point)

Let $A[1 \ldots n]$ be an array storing pairwise distinct numbers.

What is the running time of the fastest algorithm that returns a number $x$, such that at least $n/3$ numbers in $A$ are less than $x$ and at least $n/3$ numbers in $A$ are larger than $x$?

- $\Theta(n \log n)$.
- $\Theta(n^2)$.
- $\Theta(1)$.
- $\Theta(n)$. 

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Question 7 (1 point)

In class, we have seen that a Heap can be visualized as a binary tree. Assume that, instead, we visualize a heap as a tree in which each node has at most four children, all levels are full, except possibly the last level, but at the last level, all leaves are as far to the left as possible.

Let $n$ be the number of nodes of this heap. What is the height of this heap?

- $\lfloor \log_4(4n) \rfloor$
- $\lfloor \log_4(3n) \rfloor$
- $\lfloor \log_3(4n) \rfloor$
- $\lfloor \log_4(n) \rfloor$
Question 8 (1 point)

In the SubSetSum problem, the input is a tuple \((a_1, a_2, \ldots, a_n, t)\) of positive integers. The question is if there exists a subset \(I\) of \(\{1, 2, \ldots, n\}\) such that \(\sum_{i \in I} a_i = t\).

For each \(k\) and \(\ell\) with \(1 \leq k \leq n\) and \(0 \leq \ell \leq t\), let the Boolean variable \(X(k, \ell)\) be TRUE if and only if there exists a subset \(I\) of \(\{1, 2, \ldots, k\}\) such that \(\sum_{i \in I} a_i = \ell\).

For each \(k\) and \(\ell\) with \(1 \leq k \leq n\) and \(\ell < 0\), let the Boolean variable \(X(k, \ell)\) be FALSE.

Which of the following is correct for \(2 \leq k \leq n\) and \(0 \leq \ell \leq t\)?

- \(X(k, \ell) = \) TRUE if and only if \(X(k - 1, \ell) = \) TRUE or \(X(k - 1, \ell - a_k) = \) TRUE.

- \(X(k, \ell) = \) TRUE if and only if \(X(k - 1, \ell) = \) TRUE or \(X(k - 1, \ell + a_k) = \) TRUE.

- \(X(k, \ell) = \) TRUE if and only if \(X(k - 1, \ell) = \) TRUE and \(X(k - 1, \ell - a_k) = \) TRUE.

- \(X(k, \ell) = \) TRUE if and only if \(X(k - 1, \ell) = \) TRUE and \(X(k - 1, \ell + a_k) = \) TRUE.
We have seen in class that in Dynamic Programming, the subproblems are solved in a bottom-up fashion, i.e., smaller subproblems are solved before larger subproblems.

What happens if we solve subproblems in a top-down fashion, i.e., to solve a subproblem, we recursively solve smaller subproblems?

- In general, this leads to a polynomial running time.
- None of the other answers.
- In general, the output of the algorithm may be wrong.
- In general, this leads to an exponential running time.
In the SubSetSum problem, the input is a tuple \((a_1, a_2, \ldots, a_n, t)\) of integers. The question is if there exists a subset \(I\) of \(\{1, 2, \ldots, n\}\) such that \(\sum_{i \in I} a_i = t\).

In the Partition problem, the input is a tuple \((b_1, b_2, \ldots, b_m)\) of integers. The question is if there exists a subset \(J\) of \(\{1, 2, \ldots, m\}\) such that \(\sum_{j \in J} b_j = \sum_{j \notin J} b_j\).

Is the following TRUE or FALSE?

Partition \(\leq_P\) SubSetSum, i.e. Partition is polynomial-time reducible to SubSetSum.

- All of the other answers.
- None of the other answers.
- FALSE
- TRUE
Let $G = (V,E)$ be a connected undirected graph. A Hamilton cycle is a cycle that visits every vertex exactly once. An Euler cycle is a cycle that visits every edge exactly once.

Consider the following function $f$: It takes as input a connected undirected graph $G = (V,E)$. The function $f$ converts $G$ to the graph $f(G)$ with vertex set $E$. For any two $e$ and $e'$ in $E$, the graph $f(G)$ contains the edge $\{e,e'\}$ if and only if $e$ and $e'$ have exactly one vertex in common.

Is the following TRUE or FALSE?

The graph $G$ has an Euler cycle if and only if the graph $f(G)$ has a Hamilton cycle.

- All of the other answers.
- TRUE
- None of the other answers.
- FALSE
Let $G = (V, E)$ be a directed graph in which every edge $(u, v)$ has a positive weight $\omega(u, v)$. Let $s$ and $t$ be two distinct vertices of $V$, and let $P$ be a longest path in $G$ from $s$ to $t$. Note that in a path, no vertex can be visited more than once; in other words, $P$ does not contain any cycle.

Let $x$ and $y$ be two distinct vertices on $P$, both different from $s$ and different from $t$, such that $x$ is before $y$ on $P$.

Is the following TRUE or FALSE?

The subpath of $P$ from $x$ to $y$ must be a longest path in $G$ from $x$ to $y$.

- None of the other answers.
- All of the other answers.
- TRUE
- FALSE
Question 13 (1 point)

Let $G = (V, E)$ be a connected undirected graph, in which every edge $\{u, v\}$ has a positive weight $\omega(u, v)$. Assume that all edge weights are pairwise distinct.

Consider the following algorithm:

Step 1: Split the vertex set $V$ into two non-empty and disjoint subsets $A$ and $B$.

Step 2: Recursively compute a minimum spanning tree $T_A$ of the graph $G_A = (A, \{\{u, v\} \in E : u \in A, v \in A\})$.

Step 3: Recursively compute a minimum spanning tree $T_B$ of the graph $G_B = (B, \{\{u, v\} \in E : u \in B, v \in B\})$.

Step 4: Compute the edge $\{a, b\}$ of minimum weight in $E$, with $a \in A$ and $b \in B$.

Step 5: Return the union of $T_A, T_B,$ and the edge $\{a, b\}$.

Which of the following is correct?

- **a** None of the other answers.
- **b** For every input graph $G$, this algorithm computes a minimum spanning tree of $G$.
- **c** For every input graph $G$, this algorithm does not compute a minimum spanning tree of $G$.
- **d** There exists an input graph $G$ such that this algorithm does not compute a minimum spanning tree of $G$. 

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Question 14 (1 point)

Let $G = (V, E)$ be a connected undirected graph, in which every edge $\{u, v\}$ has a positive weight $\omega(u, v)$. Assume that all edge weights are pairwise distinct.

Consider the following algorithm:

While $G$ has a cycle: Take an arbitrary cycle $C$ in $G$, take the edge $\{u, v\}$ on $C$ that has maximum weight. Set $G = (V, E \setminus \{\{u, v\}\})$.

Which of the following is correct?

- [ ] This algorithm can be implemented such that its running time is linear in the number of vertices in $G$.
- [ ] This algorithm can be implemented such that its running time is polynomial in the number of vertices in $G$.
- [ ] None of the other answers.
- [ ] The number of cycles in $G$ can be exponential in the number of vertices. Therefore, the worst-case running time of this algorithm is exponential in the number of vertices.
Let $G = (V, E)$ be a connected undirected graph, in which every edge $\{u, v\}$ has a weight $\omega(u, v)$. Each edge weight can be positive or negative.

The minimum-weight spanning subgraph of $G$ is a graph $G' = (V, E')$ with $E' \subseteq E$ such that the total weight of all edges in $E'$ is minimum.

Which of the following is true?

- The minimum-weight spanning graph cannot be a tree.
- None of the other answers.
- The minimum-weight spanning graph is not necessarily a tree.
- The minimum-weight spanning graph must be a tree.
Final Exam - Preview

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Question 16 (1 point)

Let \( G = (V, E) \) be a directed acyclic graph. Consider the following algorithm:

Step 1: Run depth-first search.

Step 2: Sort the vertices in increasing order of their pre-numbers.

Which of the following is true?

- There exists a directed acyclic graph \( G = (V, E) \), for which this algorithm does not produce a topological sorting of \( G \).
- None of the other answers.
- All of the other answers.
- For any directed acyclic graph \( G \), this algorithm produces a topological sorting of \( G \).
Question 17 (1 point)

Let \( G = (V, E) \) be a directed acyclic graph.

Is the following TRUE or FALSE?

There must be a vertex in \( V \) whose out-degree is zero.

- FALSE
- All of the other answers.
- TRUE
- None of the other answers.
Let $V$ be the set of all courses that are being taught at Carleton University during the winter term of 2022.

We define an undirected graph $G = (V, E)$ with vertex set $V$. For any two vertices $u$ and $v$, the edge $\{u, v\}$ is in $E$ if and only if there is at least one student who takes both courses $u$ and $v$.

Consider a sequence $t_1, t_2, \ldots, t_k$ of $k$ pairwise disjoint time slots. In an exam schedule for this sequence, each course in $V$ is assigned to exactly one time slot. The exam schedule is conflict-free, if every student writes their exams in different time slots.

Assume that the graph $G$ is bipartite.

What is the minimum value of $k$, such that a conflict-free exam schedule with $k$ time slots is possible?

- $2$
- $4$
- $5$
- $3$
Let $G = (V, E)$ be a directed acyclic graph in which every edge $(u, v)$ has a weight $\omega(u, v)$. Assume that the smallest edge weight is $-2022$.

Let $G' = (V, E)$ be the graph with the same vertex and edge sets as $G$, but in which every edge $(u, v)$ has weight $\omega'(u, v) = \omega(u, v) + 2023$. Note that all edge weights in $G'$ are positive.

Let $s$ and $t$ be two distinct vertices in $V$, and consider a path $P = (s, u_1, u_2, \ldots, u_k, t)$ in $G$ from $s$ to $t$. Note that $P$ is also a path from $s$ to $t$ in $G'$.

Which of the following is true?

- If $P$ is a shortest path in $G'$ from $s$ to $t$, then $P$ cannot be a shortest path in $G$ from $s$ to $t$.
- None of the other answers.
- If $P$ is a shortest path in $G'$ from $s$ to $t$, then $P$ is not necessarily a shortest path in $G$ from $s$ to $t$.
- If $P$ is a shortest path in $G'$ from $s$ to $t$, then $P$ is also a shortest path in $G$ from $s$ to $t$. 

Let $G = (V, E)$ be a connected undirected graph, in which every edge $\{u, v\}$ has a positive weight $\omega(u, v)$. Assume that all edge weights are pairwise distinct.

Consider the following algorithm:

While $G$ has a cycle: Take an arbitrary cycle $C$ in $G$, take the edge $\{u, v\}$ on $C$ that has maximum weight. Set $G = (V, E \setminus \{\{u, v\}\})$.

Which of the following is correct?

- None of the other answers.
- After termination, $G$ is a minimum spanning tree of the original input graph.
- After termination, $G$ is not necessarily a minimum spanning tree of the original input graph.
- The algorithm is flawed, because it may not even terminate.
Question 21 (1 point)

Assume you are given an algorithm that decides, for any undirected graph $G$ with $n$ vertices and $m$ edges, whether or not $G$ contains a Hamilton cycle. The running time of this algorithm is $O((n + m)^{2022})$.

Is it true that every problem in NP can be solved in $O(N^{2023})$ time, where $N$ is the length of the input?

- None of the other answers.
- This is true.
- This is not true.
- All of the other answers.
Is the following claim TRUE or FALSE?

Let $L$ and $L'$ be two decision problems. Assume that $L$ is in NP, $L \leq_P L'$, and $L'$ is NP-complete. Then $L$ is also NP-complete.

- TRUE
- FALSE
- All of the other answers.
- None of the other answers.
What does NP stand for?

- No problem.
- Nurse practitioner.
- Non-polynomial time.
- Non-deterministic polynomial time.

Submit Quiz

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In the HamiltonCycle problem, the input is an undirected graph $G$, and the question is if $G$ contains a Hamilton cycle. Note that the answer is either YES or NO.

In the NonHamiltonCycle problem, the input is an undirected graph $G$, and the question is if $G$ does not contain a Hamilton cycle. Again, the answer is either YES or NO.

Is the following statement TRUE or FALSE?

In class, we have seen a proof that HamiltonCycle is in NP. By swapping YES and NO in this proof, this proves that NonHamiltonCycle is also in NP.

- FALSE
- None of the other answers.
- TRUE
- All of the other answers.
Final Exam - Preview

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Question 25 (1 point)

In the SubSetSum problem, the input is a tuple $(a_1, a_2, \ldots, a_n, t)$ of integers. The question is if there exists a subset $I$ of $\{1, 2, \ldots, n\}$ such that $\sum_{i \in I} a_i = t$.

Assume you are given a decision algorithm $A$ that correctly returns YES or NO on any such input. Note that $A$ only returns YES or NO; it does not return anything else.

Consider the following algorithm $B$ that takes as input a tuple $(a_1, a_2, \ldots, a_n, t)$ of integers.

Step 1: Set $I = \emptyset$.

Step 2: For $i = 1, 2, \ldots, n$: If $A(a_1, \ldots, a_{i-1}, a_i+1, \ldots, a_n, t)$ returns NO: Add $i$ to the set $I$.

Step 3: Return $I$.

Is the following statement TRUE of FALSE?

If there exists a subset $I$ of $\{1, 2, \ldots, n\}$ such that $\sum_{i \in I} a_i = t$, then algorithm $B$ returns such a subset.

- a: TRUE
- b: None of the other answers.
- c: All of the other answers.
- d: FALSE