Correctness:

Claim: After `explore(v)` has terminated: For every vertex `u`,
\[ \text{visited}(u) = \text{true} \iff \exists \text{ path from } v \text{ to } u. \]

Proof: \(\Rightarrow\) follows from the algorithm: the algorithm
"walks" from a vertex to a neighboring vertex.

\(\Leftarrow\) Assume \(\exists \text{ path from } v \text{ to } u.\)

Consider an arbitrary path from \(v\) to \(u\).
Assume at termination, \(\text{visited}(u) = \text{false}.\)

Let \(z\) be the last vertex on this path for which
\[ \text{visited}(z) = \text{true} \text{ at termination}. \]
Let \(w\) be the vertex on this path after \(z\).
Note: At termination, \(\text{visited}(w) = \text{false}\)

\[ (\text{true} \quad \text{true} \quad \text{false} \quad \text{false}) \]

At the moment when \(\text{visited}(z)\) is set to true,
the call `explore(z)` generates the call `explore(w)`,
during which \(\text{visited}(w)\) is set to true.

\[ \square \]

Q.E.D.
Undirected graph $G = (V, E)$.

How to compute the connected components of $G$?

At termination:

number the connected components as $1, 2, 3, ...$

for each vertex $v$:

$cc\ number\ (v) = \text{number of the connected component that vertex } v \text{ belongs to.}$
Algorithm DFS(G): // depth-first search

for all \( v \in V \): \( \text{visited}(v) = \text{false} \);
\( cc = 0 \);
for all \( v \in V \):
  if \( \text{visited}(v) = \text{false} \):
    \( cc = cc + 1 \);
    \( \text{explore}(v) \)

In algorithm \( \text{explore}(v) \):
(see page 68)

\( \text{previsit}(v) \equiv \text{"ccnumber}(v) = cc" \)

\( \text{postvisit}(v) \equiv \text{"empty"} \)

Run DFS(G) on:

![Graph](image)

Assume: adjacency lists are sorted alphabetically.
The result is the following DFS-forest:

Running time of DFS:
* first for-loop: $O(|V|)$
* second for-loop:
  - `explore(u)` is called exactly once for each vertex $u$ (this may be part of a recursive call)
  - time spent for `explore(u)`, excluding recursive calls, is $O(1 + \text{degree}(u))$. 
Total time:

\[ \mathcal{O} \left( |V| + \sum_{u \in V} (1 + \text{degree}(u)) \right) \]

\[ = \mathcal{O}( |V| + |E| ) \]