Midterm COMP 3804

February 25, 2008

Student Name:

Student Number:

35 marks total

Question 1: (5+10 marks) Let $A[1 \dots n]$ be an array containing n distinct real numbers. This array has the property that $A[1] > A[2]$ and $A[n-1] < A[n]$. We say that the array $A[1 \dots n]$ has a *local minimum* if there exists an integer i with $2 \leq i \leq n-1$ such that

$$A[i] < A[i-1] \text{ and } A[i] < A[i+1].$$

(1.1) Prove that the array $A[1 \dots n]$ has a local minimum.

(1.2) Describe an algorithm in plain English that computes, in $O(\log n)$ time, a local minimum in $A[1 \dots n]$.

Question 2: (15 marks) Let $A[1 \ldots n]$ be an array containing n distinct real numbers (some of these may be negative).

(2.1) Describe an algorithm in plain English that computes, in $O(n)$ time, an index j for which the sum

$$A[1] + A[2] + \ldots + A[j]$$

is maximum.

(2.2) Describe a divide-and-conquer algorithm in plain English that computes, in $O(n \log n)$ time, two indices i and j with $i \leq j$ for which the sum

$$A[i] + A[i + 1] + \ldots + A[j]$$

is maximum.

Question 3: (5 marks) Let $G = (V, E)$ be a connected and undirected graph, in which each edge has a weight, which is a positive real number. You may assume that no two edges have the same weight.

Prove that the edge with the second smallest weight is an edge in the minimum spanning tree of G. (*Hint:* Think of Kruskal's algorithm.)