

COMP 3804 — Winter 2026 — Problem Set 6

Some useful facts:

1. $1 + 2 + 3 + \cdots + n = n(n + 1)/2$.
2. for any real number $x > 0$, $x = 2^{\log x}$.
3. For any real number $x \neq 1$ and any integer $k \geq 1$,

$$1 + x + x^2 + \cdots + x^{k-1} = \frac{x^k - 1}{x - 1}.$$

4. For any real number $0 < \alpha < 1$,

$$\sum_{i=0}^{\infty} \alpha^i = \frac{1}{1 - \alpha}.$$

Master Theorem:

1. Let $a \geq 1$, $b > 1$, $d \geq 0$, and

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ a \cdot T(n/b) + \Theta(n^d) & \text{if } n \geq 2. \end{cases}$$

2. If $d > \log_b a$, then $T(n) = \Theta(n^d)$.
3. If $d = \log_b a$, then $T(n) = \Theta(n^d \log n)$.
4. If $d < \log_b a$, then $T(n) = \Theta(n^{\log_b a})$.

Question 1: The *set cover problem* is defined as follows:

SETCOVER = $\{(B, S_1, S_2, \dots, S_m, K) :$ B is a finite set; m is an integer;
 S_1, S_2, \dots, S_m are sets with $\cup_{i=1}^m S_i = B$;
 K is an integer;
there exists a subset $I \subseteq \{1, 2, \dots, m\}$ of
size K , such that $\cup_{i \in I} S_i = B \}$.

Prove that the decision problem SETCOVER is in **NP**.

Question 2: *Los Tabernacos* is a famous poutine restaurant in Playa del Carmen, Mexico. The owners want to advertize their restaurant to all people (“users”) on Instagram. For a given integer K , they ask K users to post a picture of the restaurant¹ on their account.

All users follow the Instagram etiquette: If user u posts a picture, then all users who follow u post a copy of this picture.

Can the owners of Los Tabernacos choose K users such that all Instagram users post a picture of the restaurant?

- Formulate this problem as a decision problem LOSTABERNACOS on a graph.
- Prove that $\text{LOSTABERNACOS} \leq_P \text{SETCOVER}$, i.e., in polynomial time, LOSTABERNACOS can be reduced to SETCOVER.

¹and offer them free poutine