Carleton University
COMP/MATH 3804 A/B, Test 2
Solutions

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Each of the five questions is worth 10 marks.

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Algorithm DFS(G):
for each vertex u
do visited(u) = false
endfor;
cc = 0;
for each vertex v        (*)
do if visited(v) = false
    then cc = cc + 1
         EXPLORE(v)
    endif
endfor
```

```
Algorithm EXPLORE(v):
visited(v) = true;
ccnumber(v) = cc;
for each edge {v, u}     (*)
do if visited(u) = false
    then EXPLORE(u)
    endif
endfor
```

Question 1: Professor Taylor Swift claims that she has proved the following:

Professor Swift's Claim: Let S be a set of n distinct numbers, and assume that these numbers are stored in a min-heap. Then we can obtain the elements of S in sorted order in $O(n)$ time.

Is Professor Swift's Claim correct? As always, justify your answer.

Solution: Swifties will be disappointed: Taylor's Claim is wrong!
We prove this by contradiction. Assume the claim is true.

- We have seen in class that a min-heap for the numbers in S can be constructed in $O(n)$ time.
- By the assumption, we can use this min-heap to sort S in $O(n)$ time.
- Thus, we can sort any set of n numbers in $O(n)$ time. This is a contradiction, because sorting has an $\Omega(n \log n)$ lower bound.

Question 2: Let S be a set of n distinct numbers, where $n \geq 2026$. Assume this set S is stored in a min-heap $A(1 \dots n)$. Let x be the fourth smallest number in S. What is the set of all possible indices i such that x may be stored in $A(i)$? As always, justify your answer.

Solution: To read the solution, you should draw the binary tree representing the min-heap. We may assume that $S = \{1, 2, 3, \dots, n\}$, so that $x = 4$.

- x cannot be stored at index 1, because the smallest number, which is 1, is stored there.
- x can be stored at index 2: Index 1 stores 1, index 2 stores 4, index 3 stores 2, index 6 stores 3. By a symmetric argument, x can be stored at index 3.
- x can be stored at index 4: Index 1 stores 1, index 2 stores 2, index 4 stores 4, index 3 stores 3. By a symmetric argument, x can be stored at any of the indices 5, 6, and 7.
- x can be stored at index 8: Index 1 stores 1, index 2 stores 2, index 4 stores 3, index 8 stores 4. By a symmetric argument, x can be stored at any of the indices $9, \dots, 15$.
- x cannot be stored at any index ≥ 16: Assume it is stored at such an index. The path from the parent of this index to the root has at least four nodes, and each of them stores a number that is smaller than x.
- The set of all possible indices is

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}.$$

Question 3: Does there exist an undirected graph that has 85 vertices, such that exactly 21 of these vertices have degree 7, and the other 64 vertices have degree 8? As always, justify your answer.

Solution: We have seen in class that the sum of all degrees is equal to twice the number of edges. In particular, this sum is even. For the graph in this question, the sum of all degrees is

$$21 \cdot 7 + 64 \cdot 8,$$

which is odd. Thus, the graph in this question does not exist.

Question 4: Let G be an undirected connected graph, and let A be a vertex of G. We run algorithm EXPLORE(A) on this graph.

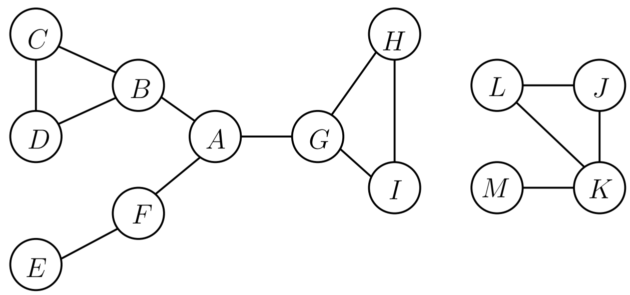
Is the following true or false?

If there is a back edge, then the graph G cannot be bipartite.

As always, justify your answer.

Solution: The claim is false. Here is a counter-example. Let G be the cycle (A, B, C, D, A). If we run EXPLORE(A), then $\{A, B\}$, $\{B, C\}$, and $\{C, D\}$ are tree edges, whereas $\{D, A\}$ is a back edge. However, the graph G is bipartite: Place A and C in L, and place B and D in R. Then every edge is between a vertex in L and a vertex in R.

Question 5: Consider the following undirected graph:



Draw the DFS-forest obtained by running algorithm DFS on this graph. (Just draw the DFS-forest, no explanation is needed.)

In the forest, draw each tree edge as a solid edge, and draw each back edge as a dotted edge.

Whenever there is a choice of vertices (see the two lines labeled (*) on page 1), pick the one that is alphabetically first.

Solution: