Carleton University
COMP/MATH 3804 A/B, Test 2

February 11, 2026

Student Name:

Student Number:

Each of the five questions is worth 10 marks.

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Algorithm DFS(G):
for each vertex u
do visited(u) = false
endfor;
cc = 0;
for each vertex v        (*)
do if visited(v) = false
    then cc = cc + 1
         Explore(v)
    endif
endfor
```

```
Algorithm Explore(v):
visited(v) = true;
ccnumber(v) = cc;
for each edge {v, u}     (*)
do if visited(u) = false
    then Explore(u)
    endif
endfor
```

Question 1: Professor Taylor Swift claims that she has proved the following:

Professor Swift's Claim: Let S be a set of n distinct numbers, and assume that these numbers are stored in a min-heap. Then we can obtain the elements of S in sorted order in $O(n)$ time.

Is Professor Swift's Claim correct? As always, justify your answer.

Question 2: Let S be a set of n distinct numbers, where $n \geq 2026$. Assume this set S is stored in a min-heap $A(1 \ldots n)$. Let x be the fourth smallest number in S. What is the set of all possible indices i such that x may be stored in $A(i)$? As always, justify your answer.

Question 3: Does there exist an undirected graph that has 85 vertices, such that exactly 21 of these vertices have degree 7, and the other 64 vertices have degree 8? As always, justify your answer.

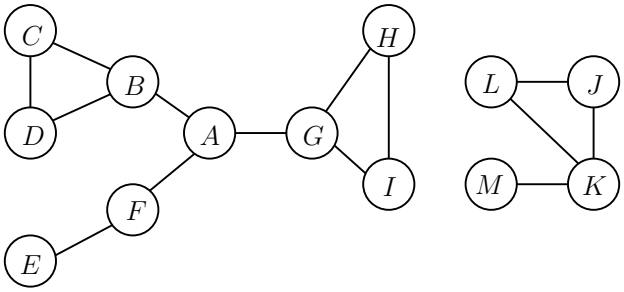
Question 4: Let G be an undirected connected graph, and let A be a vertex of G. We run algorithm EXPLORE(A) on this graph.

Is the following true or false?

If there is a back edge, then the graph G cannot be bipartite.

As always, justify your answer.

Question 5: Consider the following undirected graph:



Draw the DFS-forest obtained by running algorithm DFS on this graph. (Just draw the DFS-forest, no explanation is needed.)

In the forest, draw each tree edge as a solid edge, and draw each back edge as a dotted edge.

Whenever there is a choice of vertices (see the two lines labeled (*) on page 1), pick the one that is alphabetically first.