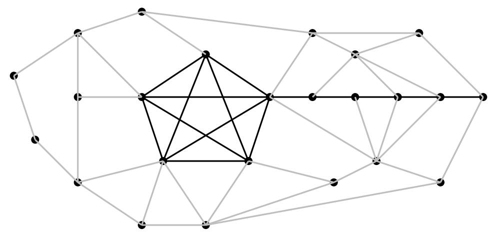
COMP 3804 — Tutorial April 5

Question 1: Let $K \geq 3$ be an integer. A K-*kite* is a graph consisting of a clique of size K and a path with K vertices that is connected to one vertex of the clique; thus, the number of vertices is equal to $2K$. In the figure below, the graph with the black edges forms a 5-kite.



The *kite problem* is defined as follows:

$$\text{KITE} = \{(G, K) : \text{ graph } G \text{ contains a } K\text{-kite}\}.$$

Prove that the language KITE is in **NP**.

Question 2: The *clique problem* is defined as follows:

$$\text{CLIQUE} = \{(G, K) : \text{ graph } G \text{ contains a clique of size } K\}.$$

Prove that CLIQUE $\leq_P$ KITE, i.e., in polynomial time, CLIQUE can be reduced to KITE.

Question 3: The *subset sum problem* is defined as follows:

$$\text{SUBSETSUM} = \{(S, t) : \; S \text{ is a set of integers, } t \text{ is an integer, } \exists S' \subseteq S \text{ such that } \textstyle\sum_{x \in S'} x = t \}.$$

The *partition problem* is defined as follows:

$$\text{PARTITION} = \{S : \; S \text{ is a set of integers, } \exists S' \subseteq S \text{ such that } \textstyle\sum_{x \in S'} x = \sum_{y \in S \setminus S'} y \}.$$

- Prove that SUBSETSUM $\leq_P$ PARTITION, i.e., in polynomial time, SUBSETSUM can be reduced to PARTITION.
- Prove that PARTITION $\leq_P$ SUBSETSUM, i.e., in polynomial time, PARTITION can be reduced to SUBSETSUM.

Question 4: The *clique and independent set problem* is defined as follows:

$$\text{CLIQUEINDEPSET} = \{(G, K) : \; \text{graph } G \text{ contains a clique of size } K \text{ and } G \text{ contains an independent set of size } K \}.$$

Prove that CLIQUE $\leq_P$ CLIQUEINDEPSET, i.e., in polynomial time, CLIQUE can be reduced to CLIQUEINDEPSET.

Question 5: Let φ be a Boolean formula in the variables $x_1, x_2, \ldots, x_n$. We say that φ is in *conjunctive normal form* (CNF) if it is of the form

$$\varphi = C_1 \wedge C_2 \wedge \ldots \wedge C_m,$$

where each C_i, $1 \leq i \leq m$, is of the following form:

$$C_i = l_1^i \vee l_2^i \vee \ldots \vee l_{k_i}^i.$$

Each l_j^i is a *literal*, which is either a variable or the negation of a variable.

The *satisfiability problem* is defined as follows:

$$\textsc{Sat} = \{\varphi : \varphi \text{ is in CNF-form and is satisfiable}\}.$$

Prove that $\textsc{Clique} \leq_P \textsc{Sat}$, i.e., in polynomial time, $\textsc{Clique}$ can be reduced to $\textsc{Sat}$.