Question 1: Let $K \geq 3$ be an integer. A $K$-kite is a graph consisting of a clique of size $K$ and a path with $K$ vertices that is connected to one vertex of the clique; thus, the number of vertices is equal to $2K$. In the figure below, the graph with the black edges forms a 5-kite.

The *kite problem* is defined as follows:

\[ \text{Kite} = \{(G, K) : \text{graph } G \text{ contains a } K\text{-kite}\}. \]

Prove that the language Kite is in \textbf{NP}.

Question 2: The *clique problem* is defined as follows:

\[ \text{Clique} = \{(G, K) : \text{graph } G \text{ contains a clique of size } K\}. \]

Prove that \text{Clique} \leq_p \text{Kite}, i.e., in polynomial time, \text{Clique} can be reduced to Kite.

Question 3: The *subset sum problem* is defined as follows:

\[ \text{SubsetSum} = \{(S, t) : S \text{ is a set of integers, } t \text{ is an integer, } \exists S' \subseteq S \text{ such that } \sum_{x \in S'} x = t \}. \]

The *partition problem* is defined as follows:

\[ \text{Partition} = \{S : S \text{ is a set of integers, } \exists S' \subseteq S \text{ such that } \sum_{x \in S'} x = \sum_{y \in S \setminus S'} y \}. \]

- Prove that \text{SubsetSum} \leq_p \text{Partition}, i.e., in polynomial time, \text{SubsetSum} can be reduced to Partition.
- Prove that \text{Partition} \leq_p \text{SubsetSum}, i.e., in polynomial time, \text{Partition} can be reduced to SubsetSum.

Question 4: The *clique and independent set problem* is defined as follows:

\[ \text{CliqueIndepSet} = \{(G, K) : \text{graph } G \text{ contains a clique of size } K \text{ and } G \text{ contains an independent set of size } K \}. \]

Prove that \text{Clique} \leq_p \text{CliqueIndepSet}, i.e., in polynomial time, \text{Clique} can be reduced to CliqueIndepSet.
**Question 5:** Let \( \varphi \) be a Boolean formula in the variables \( x_1, x_2, \ldots, x_n \). We say that \( \varphi \) is in **conjunctive normal form** (CNF) if it is of the form

\[
\varphi = C_1 \land C_2 \land \ldots \land C_m,
\]

where each \( C_i, 1 \leq i \leq m \), is of the following form:

\[
C_i = l_{i1} \lor l_{i2} \lor \ldots \lor l_{ik_i}.
\]

Each \( l_{ij} \) is a **literal**, which is either a variable or the negation of a variable.

The **satisfiability problem** is defined as follows:

\[
\text{SAT} = \{ \varphi : \varphi \text{ is in CNF-form and is satisfiable} \}.
\]

Prove that **CLIQUE \leq_p SAT**, i.e., in polynomial time, **CLIQUE** can be reduced to **SAT**.