COMP 2804 Discrete Structures II

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Office Hours: Wednesday 11:30-13:30.

https://patmorin.me/teaching/2804/

Grading Scheme.

4 Assignments	25%	
Mid-Term Exam	25%	- in-class
Final Exam	50%	- scheduled

Textbook: Michiel Smid. Discrete Structures for Computer Science.

Pseudo Theorem: Among any group of 6 people there are 3 mutual friends or 3 mutual strangers.

clique

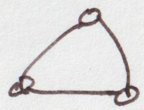
independent set.

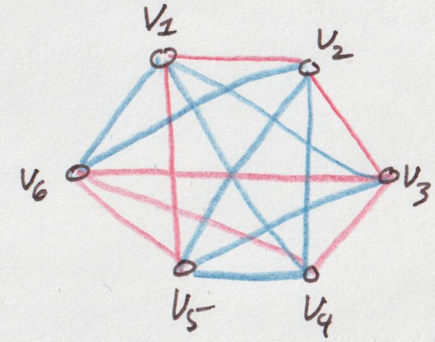
- p is a friend of q iff q is a friend of p.
- For any distinct p and q ~~either~~: exactly one of the following is true:
  - p and q are friends; OR
  - p and q are stangers.

Theorem: Let G be a complete graph with 6 vertices whose edges are coloured red and blue.

Then G contains a monochromatic 3-cycle (triangle).

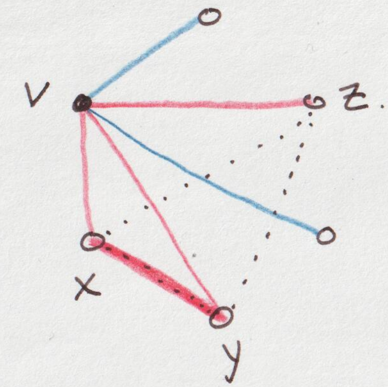
all edges the same color.





$v_6 v_4 v_3$ $v_2 v_4 v_5$

Proof: Choose any vertex v. Consider the 5 edges incident to v.

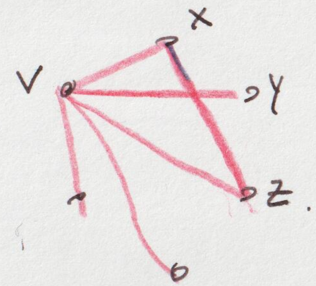


- ~~Suppose~~ Let r be the number of edges incident to v that are red, and let b the number that are blue.

Fact: $r+b=5$.

Claim: $r>2$ or $b>2$.

Proof by contradiction: If $r\le 2$ and $b \le 2$ then $r+b \le 2+2 = 4 < 5$.



→ Without loss of generality, assume $r>2$

Since r is an integer and $r>2$, $r \ge 3$.

- Let x, y, and z be vertices such that vx, vy, and vz are all red.
- Now consider the edges xy, yz, and xz.

Case (i) xy, yz, and xz are all blue. Then xyz is a monochromatic (blue) triangle.

Case (ii) [not Case (i)]: At least one of xy, yz, or xz is red. Without loss of generality assume xy is red. Then vxy is a monochromatic (red) triangle. ∎

Quicksort($a_1 \ldots a_n$). — rearange $a_1 \ldots a_n$ so that they're sorted

if $n \le 1$ return $a_1 \ldots a_n$
Pick a random i in $\{1, \ldots, n\}$.
compare $a_2 \ldots a_n$ to a_i

$<a_1$	a_1	$>a_1$
X	↑	Y

Quicksort(X) ✓
Quicksort(Y) ✓

$a_1 \; a_2 \; a_3 \quad \ldots \quad a_8$.

$\boxed{1 \; 2 \; 3 \; 4 \; 5 \; 6 \; 7 \; 8.}$

$7 = n-1$ comparison.

$\boxed{1}\;\underbrace{\boxed{2 \; 3 \; 4 \; 5 \; 6 \; 7 \; 8.}}_{|Y| = n-1.}$

X

$n-2 = 6$ comparisons.

$\boxed{2}\;\boxed{3 \; 4 \; 5 \; 6 \; 7 \; 8.}$

$\vdots$

5

4

3

2

$\boxed{7 \; 8.}$

1

$\boxed{8}$

$\cancel{8+}\, 7+6+5+4+3+2+1.$

$$n-1 + n-2 + n-3 + \cdots + 1.$$

$$\frac{n(n-1)}{2} = \frac{n^2}{2} - \frac{n}{2}.$$

n

1 000 000

$\frac{n(n-2)}{2}$

$499\,\underbrace{999\,500006}_{1 \text{ billion.}}$ comparison.

≈ 499 seconds.

$2n \ln n \approx 1.38\, n \log_2 n$

027,631,021

$\approx 0{,}027$ seconds.