Disjunction. The probability that A or B occurs is

$$\Pr\{A \cup B\} = \Pr\{A \text{ or } B\} = \Pr\{A\} + \Pr\{B\} - \Pr\{A \cap B\} \leq \Pr\{A\} + \Pr\{B\} \ .$$

The last inequality is called *Boole's inequality.* When A and B are disjoint ($A \cap B = \emptyset$), then the last inequality holds with equality (this is the third axiom of probability theory).

Expected Values. The expected value of random variable X is

$$\mathrm{E}[X] = \sum_x x \times \Pr\{X = x\}$$

where x runs over all possible values of the random variable X. Expectation has the important and incredibly useful property of *linearity*:

$$\begin{aligned} \mathrm{E}[X+Y] &= \mathrm{E}[X] + \mathrm{E}[Y] \\ \mathrm{E}\left[\sum_{i=1}^{n} X_i\right] &= \sum_{i=1}^{n} \mathrm{E}[X_i] \end{aligned}$$

Indicator Variables. Use these to count things:

$$I_i = \begin{cases} 1 & \text{if the thing we are counting happens} \\ 0 & \text{otherwise} \end{cases}$$

The expected value of indicator variables is easy to compute:

$$\mathrm{E}[I_i] = \Pr\{I_i = 1\} = \Pr\{\text{the thing we are counting happens}\}.$$

Dependence, Independence, and Conjunction. The probability that event A happens if we know that event B happens is

$$\Pr\{A \mid B\} = \Pr\{A \text{ given } B\} = \frac{\Pr\{A \cap B\}}{\Pr\{B\}} \ .$$

Rewriting the above we obtain a way of computing $\Pr\{A \cap B\}$ as

$$\Pr\{A \cap B\} = \Pr\{A \mid B\} \times \Pr\{B\} \ .$$

We say that A and B are *independent* if

$$\Pr\{A \mid B\} = \Pr\{A\} \ .$$

Dependence is symmetric, so

$$\Pr\{A \mid B\} = \Pr\{A\} \Leftrightarrow \Pr\{B \mid A\} = \Pr\{B\}$$

and all of the above are equivalent to

$$\Pr\{A \cap B\} = \Pr\{A\} \times \Pr\{B\} \ .$$

This last one turns out to be the most useful formulation of independence.